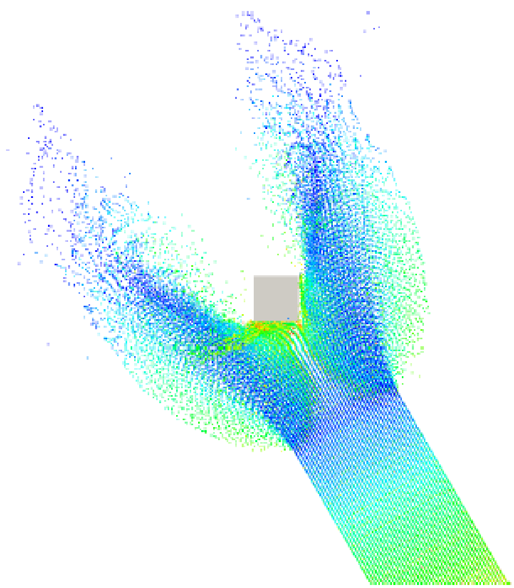
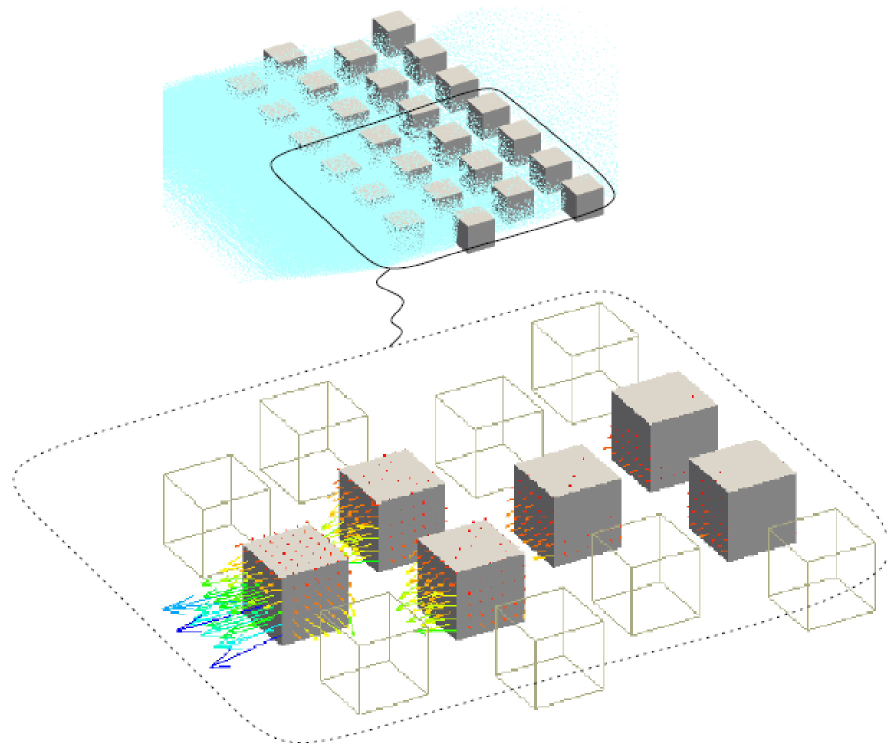
Simulating Granular Flow Dynamics and Other Applications using the Material Point Method





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7th MPM Workshop – University of Utah

Salt Lake City, Utah

March 14-15, 2013

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- ***Participants***
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    - Current Ph.D. student
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    - Ph.D. at UW (2009)
  - John A. Moore
    - MSE at UW (Spring 2007)
  - Pedro Arduino
  - Peter Mackenzie-Helnwein
  - Greg Miller

- ***Acknowledgements***
  - NSF–CMMI
  - UW-RRF

Research Goals

- Motivation
  - Looking at the big picture
    - As civil engineers, we are interested in designing, building, and maintaining critical infrastructure
      - Resiliency and sustainability
        - Long term behavior
        - Response to disaster
    - Must rely heavily on models
      - Physical based
      - Numerical based
    - Numerical simulations are a critical component for evaluating the resiliency and sustainability of critical infrastructure

Research Goals

- Motivation

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Research Goals

- Motivation
  - Looking at the big picture
    - What are the short and long term effects of natural disasters on civil infrastructure?
      - Earthquakes
      - Tsunamis
      - Landslides
      - Debris flows
    - The ability to answer this question is directly linked to our ability model these events as well as their interaction with the built environment

Research Goals

- Motivation
  - Landslides and Debris Flows
    - Highly dynamic
    - Composed of several materials
    - Can exhibit both solid-like and fluid-like behavior

Research Goals

- Goals
  - Establish a computational framework
    - Unified approach for modeling fluids and solids
    - Capture behavior associated with multiple phases
      - Mixing and separation
      - Mechanical behavior
  - Applications
    - Landslide and debris flows
      - Interaction with protective structures
    - Other areas of engineering

Mechanical behavior

- Solid phase
  - Large deformation
  - History dependent
  - Finite strains
  - Material failure

$$\boldsymbol{\sigma} = \rho \frac{\partial \psi(\boldsymbol{\varepsilon}, \boldsymbol{\xi})}{\partial \boldsymbol{\varepsilon}}$$

$$L_v \boldsymbol{\sigma} = \rho \,\overline{\mathbf{c}}(\boldsymbol{\varepsilon}, \boldsymbol{\xi}) : L_v \boldsymbol{\varepsilon}$$

$$f(\boldsymbol{\sigma}, \boldsymbol{\xi}) \leq 0 \quad \lambda \geq 0 \quad f \cdot \lambda = 0$$

- Fluid phase
  - Rate dependent but no history
  - Large deformation
  - Nearly incompressible

$$\boldsymbol{\sigma} = p\mathbf{1} + 2\mu \nabla^s \mathbf{v}$$

$$\operatorname{div} \mathbf{v} = 0$$

Material Point Method (MPM)

- To reduce computational expense …
  - Use a regular, rectangular grid.
    - Eliminates need for cell search algorithms
    - Mapping between global and local coordinates is easily accomplished
    - Allows for dynamic node/cell creation and deletion
    - Problematic for representing general surface geometry
      - We developed two distinct approaches for incorporating general boundary geometry
  - Use linear shape functions.
    - Cheapest option for rectangular cells/elements
    - Widely used
    - Leads to kinematic locking.

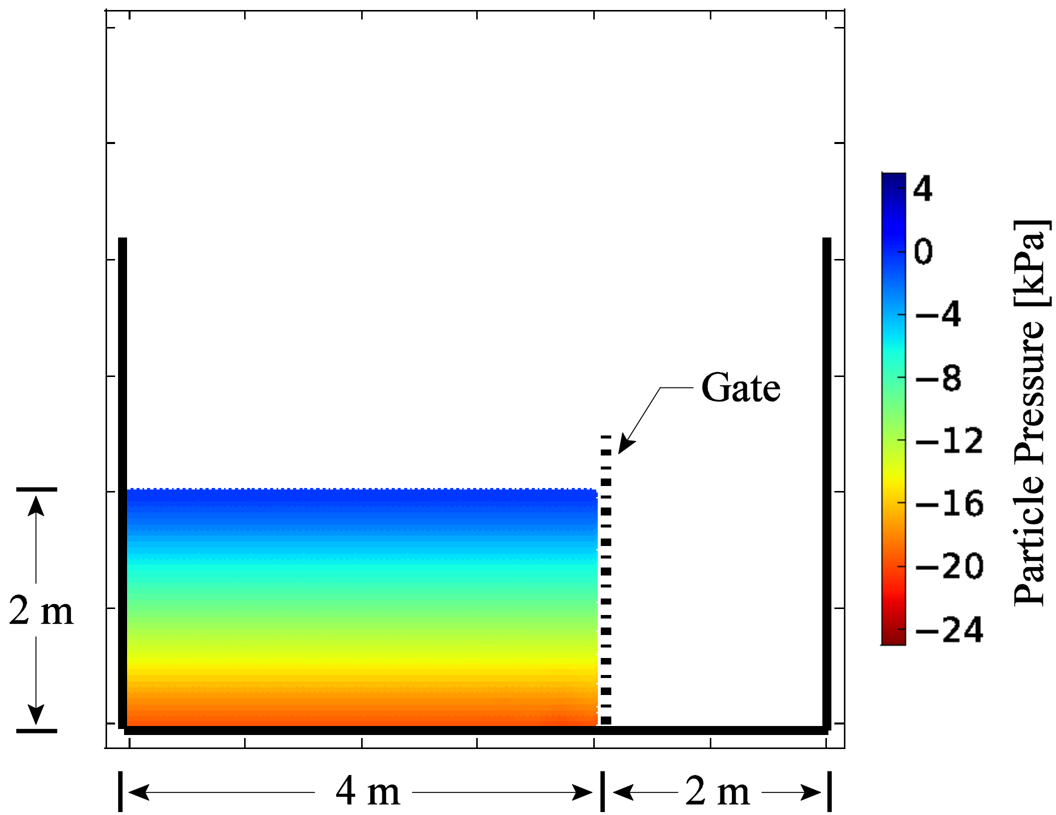
Anti-Locking Strategies in the MPM

Anti-Locking Strategies in the MPM

- Kinematic Locking
  - ‘Locking’ refers to the build up of fictitious stiffness)
  - A result of a cell’s/element’s inability to reproduce the correct mode shapes
  - We are concerned with two types of locking:
    - Volumetric Locking
      - Dominant in the incompressible limit for both solids and fluids
    - Shear Locking
      - Problematic for any material with moderate shear stiffness (or highly viscous fluids)
  - 5th and 6th MPM workshops

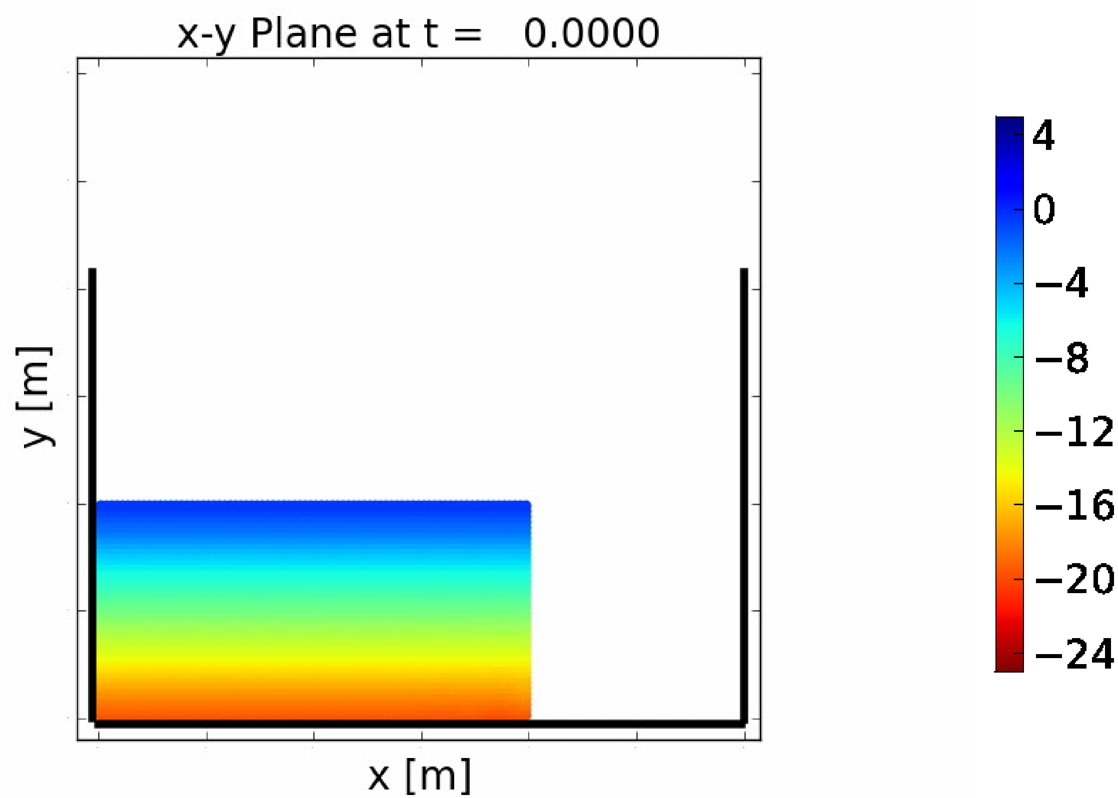
Anti-Locking Strategies in the MPM

- Kinematic Locking
  - Dam break using the Standard MPM algorithm



Anti-Locking Strategies in the MPM

- Kinematic Locking
  - Dam break using the Standard MPM algorithm
    - Particle pressure (kPa)



Anti-Locking Strategies in the MPM

- Mitigating Locking
  - Approximation functions
    - Acceleration field

$$\dot{\mathbf{v}} \approx \dot{\mathbf{v}}^h = \sum_i N_i \, \dot{\mathbf{v}}_i \qquad \delta\dot{\mathbf{v}}^h = \sum_i N_i \, \delta\dot{\mathbf{v}}_i$$

    - Strain field

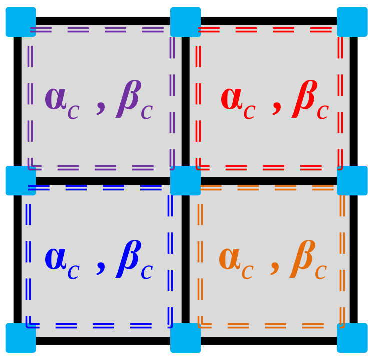
$$\varepsilon \approx \varepsilon^h = \mathbf{M}^* \, \mathbf{S} \, \alpha + [\mathbf{I} - \mathbf{M}^* \, \mathbf{M}] \, \tilde{\varepsilon} \qquad \delta\varepsilon^h = \mathbf{M}^T \, \mathbf{S} \, \delta\alpha$$

    - Stress field

$$\boldsymbol{\sigma} \approx \boldsymbol{\sigma}^h = \mathbf{M}^* \, \mathbf{S} \, \beta + [\mathbf{I} - \mathbf{M}^* \, \mathbf{M}] \, \tilde{\boldsymbol{\sigma}} \qquad \delta\boldsymbol{\sigma}^h = \mathbf{M}^T \, \mathbf{S} \, \delta\beta$$

Anti-Locking Strategies in the MPM

- Mitigating Locking
  - Control Volumes
    - Cell-based

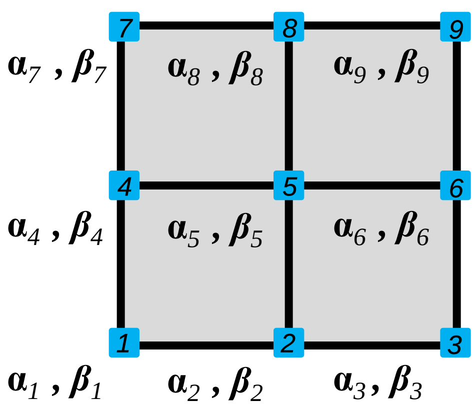


$\boldsymbol{\alpha}_c$, $\boldsymbol{\beta}_c$

$$\varepsilon_p \Longrightarrow \varepsilon_p(\boldsymbol{\alpha}_c)$$

$$\sigma_p \Longrightarrow \sigma_p(\boldsymbol{\beta}_c)$$

    - Node-based



$\boldsymbol{\alpha}_i$, $\boldsymbol{\beta}_i$

$$\varepsilon_p \Longrightarrow \sum_i N_i\, \varepsilon_i(\boldsymbol{\alpha}_i)$$

$$\sigma_p \Longrightarrow \sum_i N_i\, \sigma_i(\boldsymbol{\beta}_i)$$

- Hybrid-based

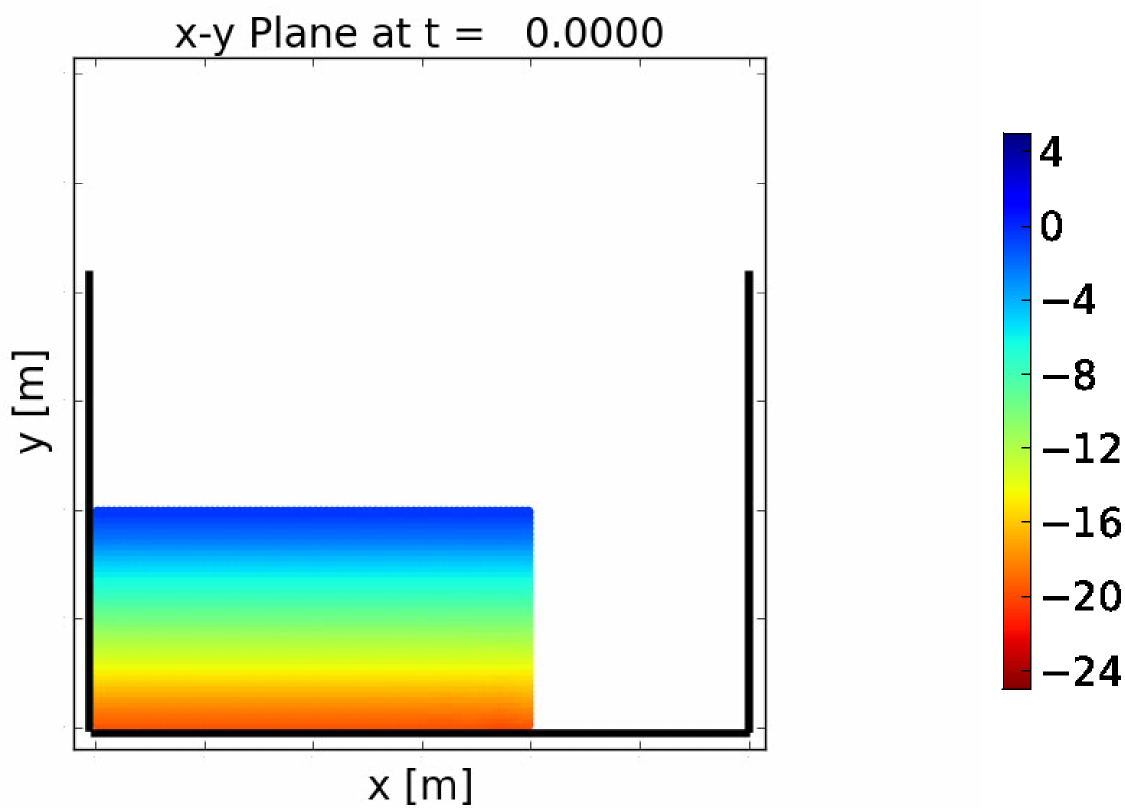
$$\varepsilon_p \Longrightarrow vol \sum_i N_i\, \varepsilon_i(\boldsymbol{\alpha}_i) + dev\ \varepsilon_p(\boldsymbol{\alpha}_c)$$

$$\sigma_p \Longrightarrow vol \sum_i N_i\, \sigma_i(\boldsymbol{\beta}_i) + dev\ \sigma_p(\boldsymbol{\beta}_c)$$

Modeling Fluid Behavior

$$\sigma = \rho\,\bar{k}\,\theta\,1 + 2\,\mu\,\mathrm{d}$$

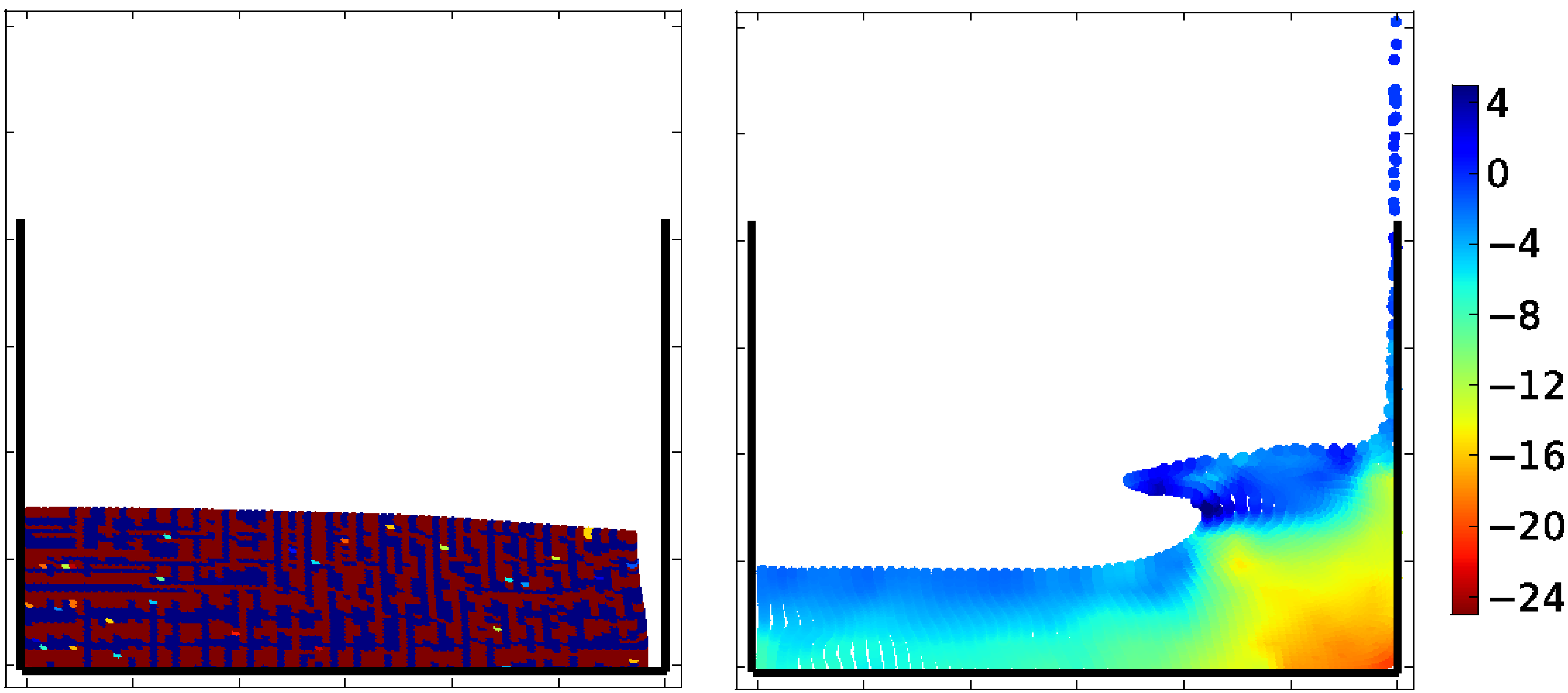
- Dam break revisited
  - Particle pressure (kPa)



Modeling Fluid Behavior

$$\sigma = \rho \bar{k} \theta \mathbf{1} + 2 \mu \mathbf{d}$$

- Dam break revisited
  - Particle pressure (kPa) at $t = 2.0\ s$

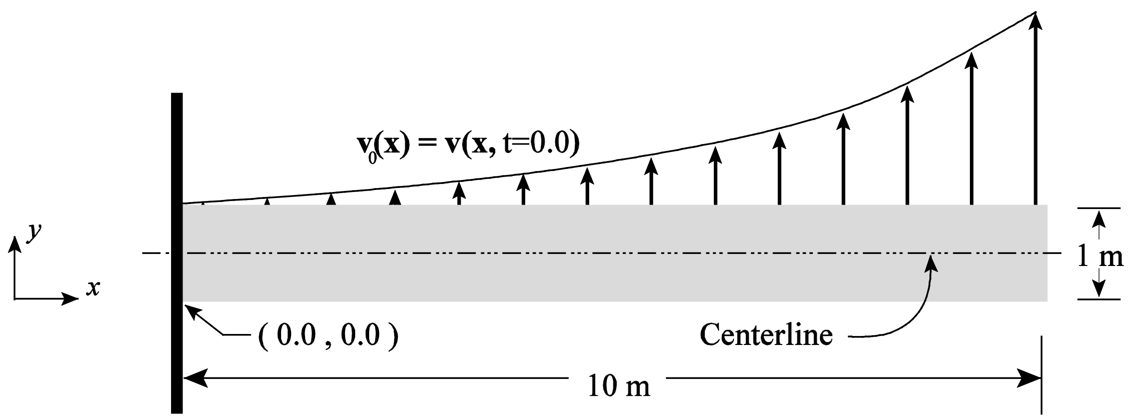


Modeling Solid Behavior

- Material Models
  - Elastic
    - Linear, nonlinear, isotropic
  - Ductile
    - J2
  - Pressure Dependent
    - Drucker Prager
    - Matusoka Nakai

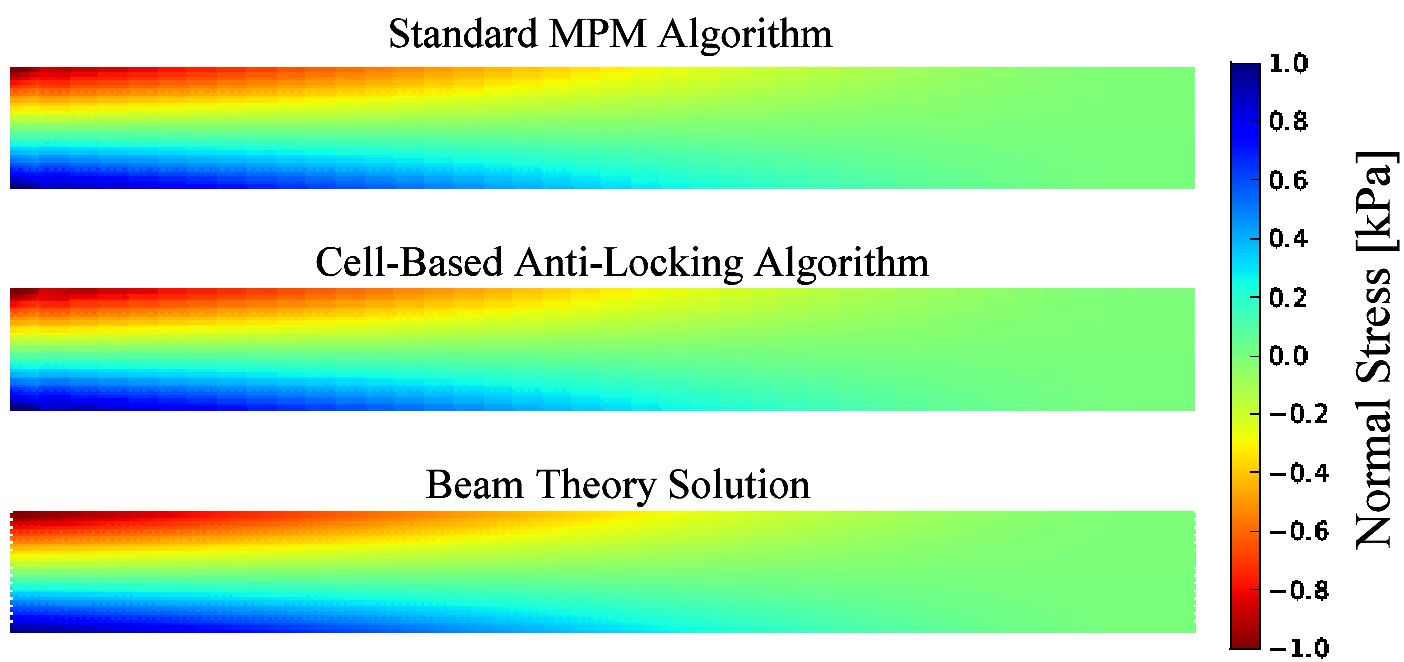
Modeling Elastic Solid Behavior

- Elastic response $\sigma = k\,\mathbf{1}\,\mathrm{tr}\,\varepsilon + 2\,G\,\mathrm{dev}\,\varepsilon$
  - Vibrating cantilever beam



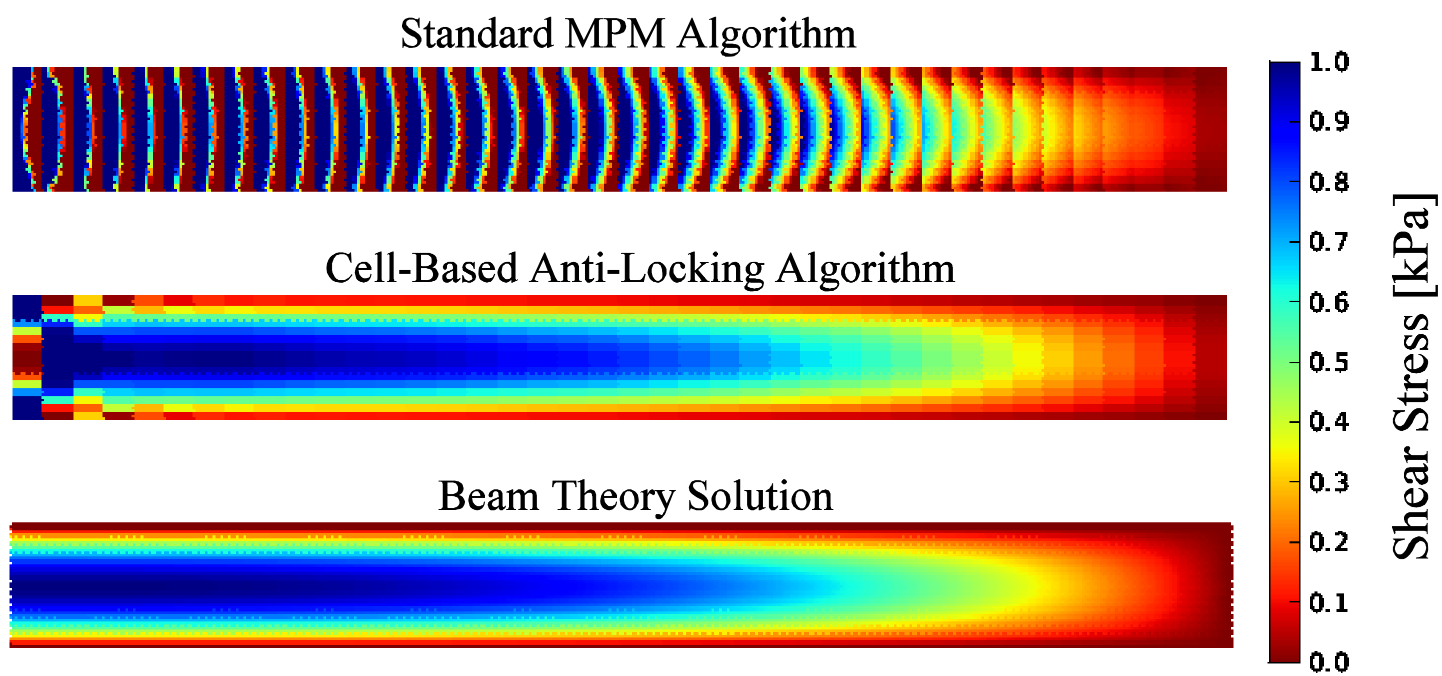
Modeling Elastic Solid Behavior

- Vibrating beam
  - Normal stress



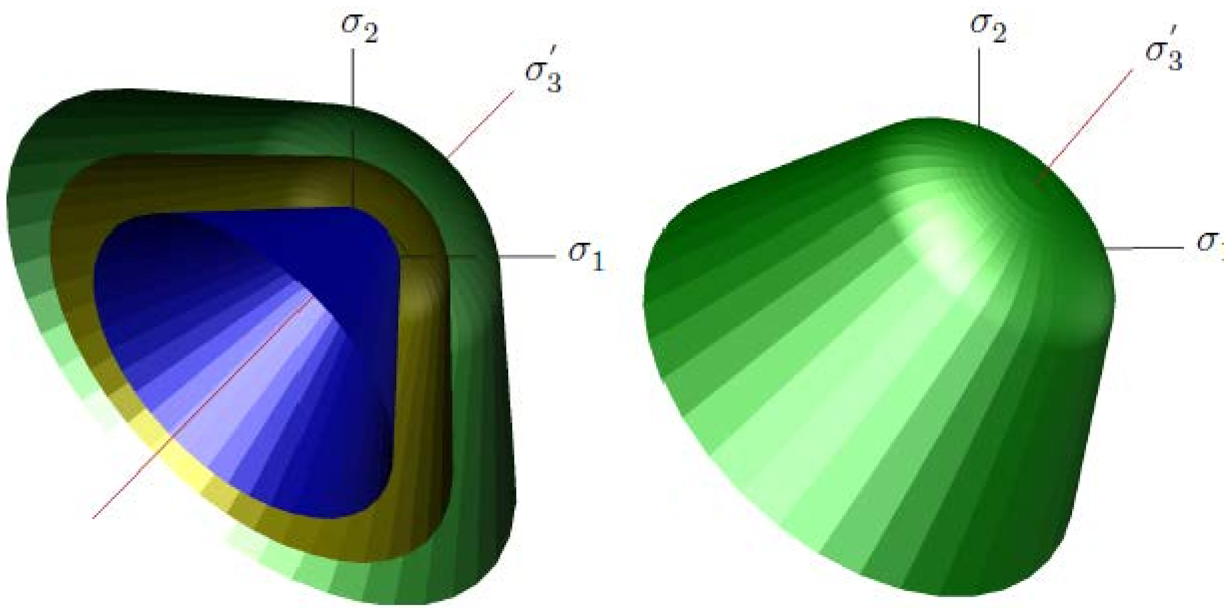
Modeling Elastic Solid Behavior

- Vibrating beam
  - Shear stress



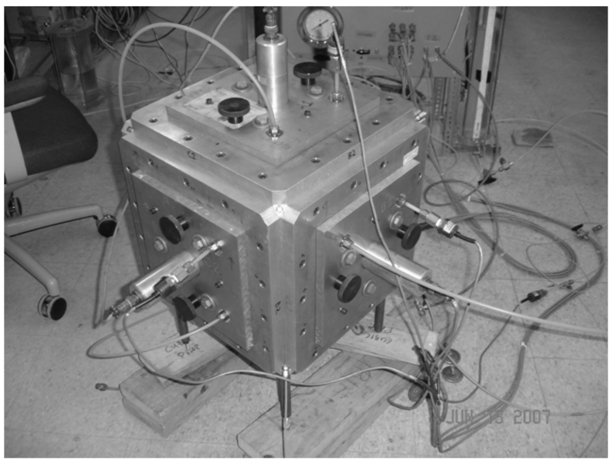
Modeling Granular Materials

- Material Models
  - Pressure Dependent
    - Drucker-Prager

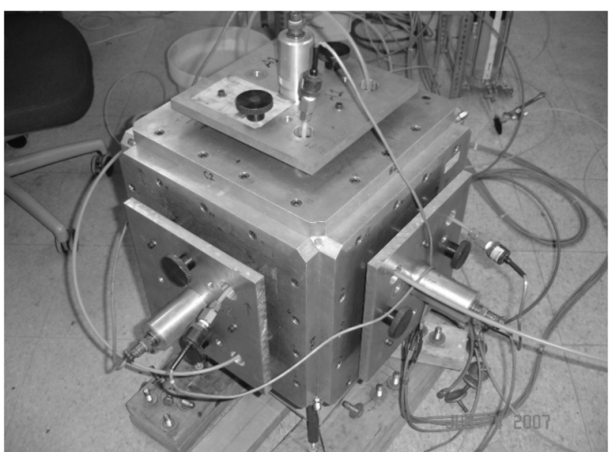


Modeling Granular Material

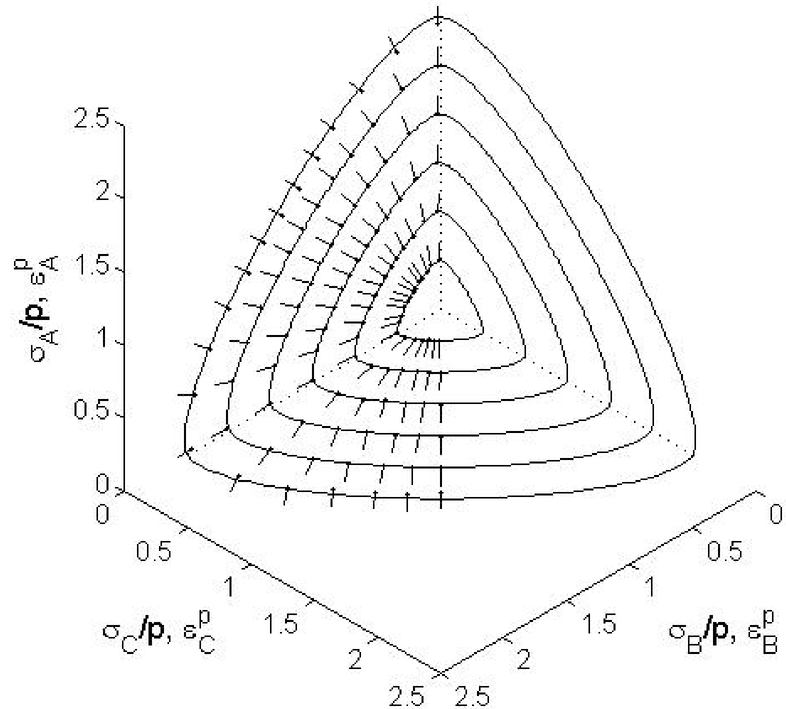
- Experimental Results



(a)

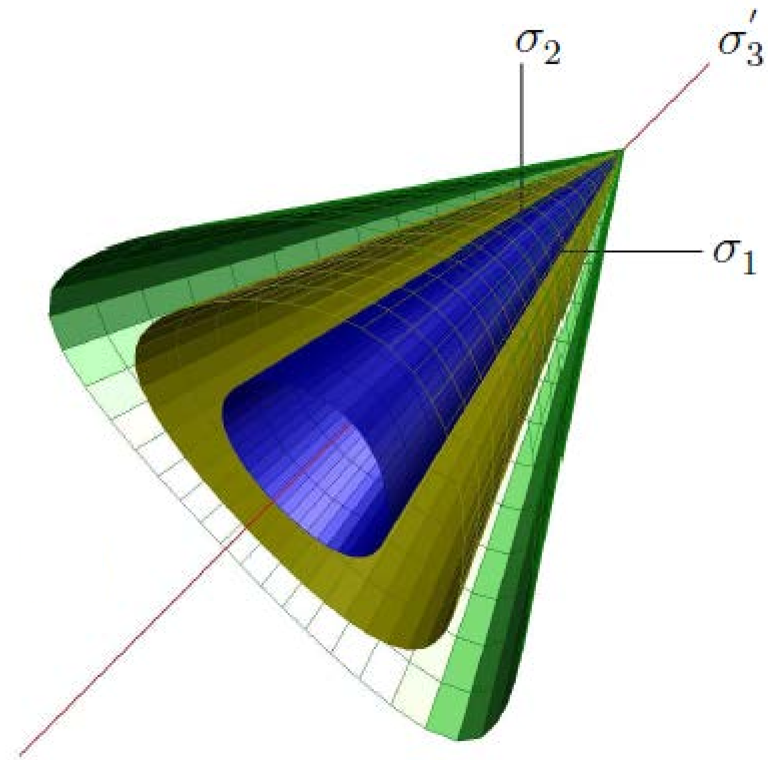


(b)



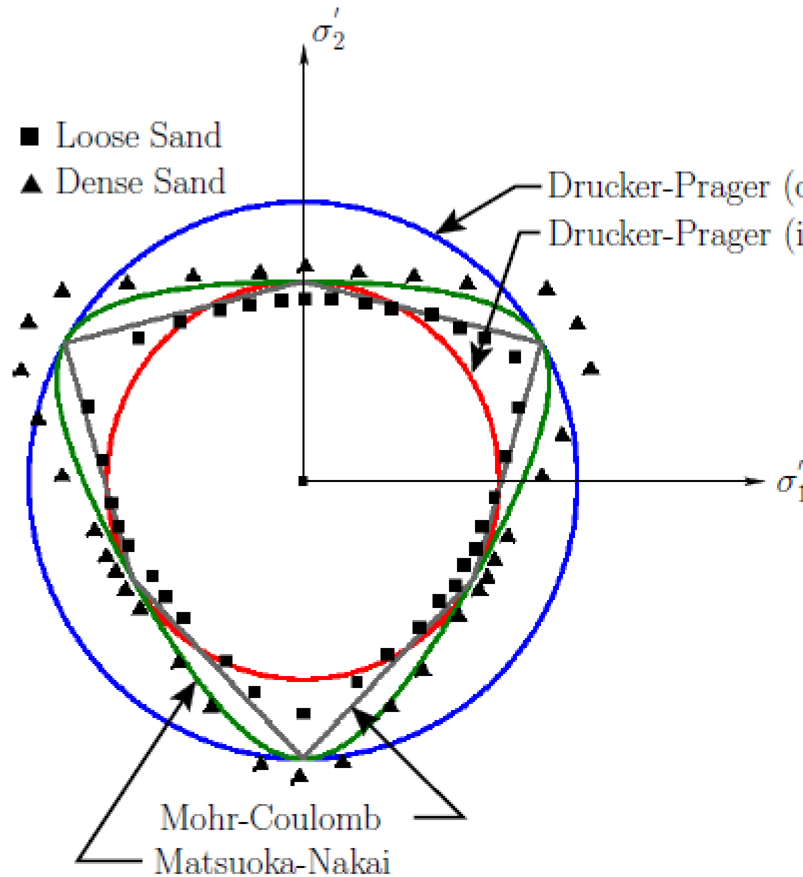
Modeling Granular Materials

- Material Models
  - Pressure Dependent
    - Matsuoka-Nakai



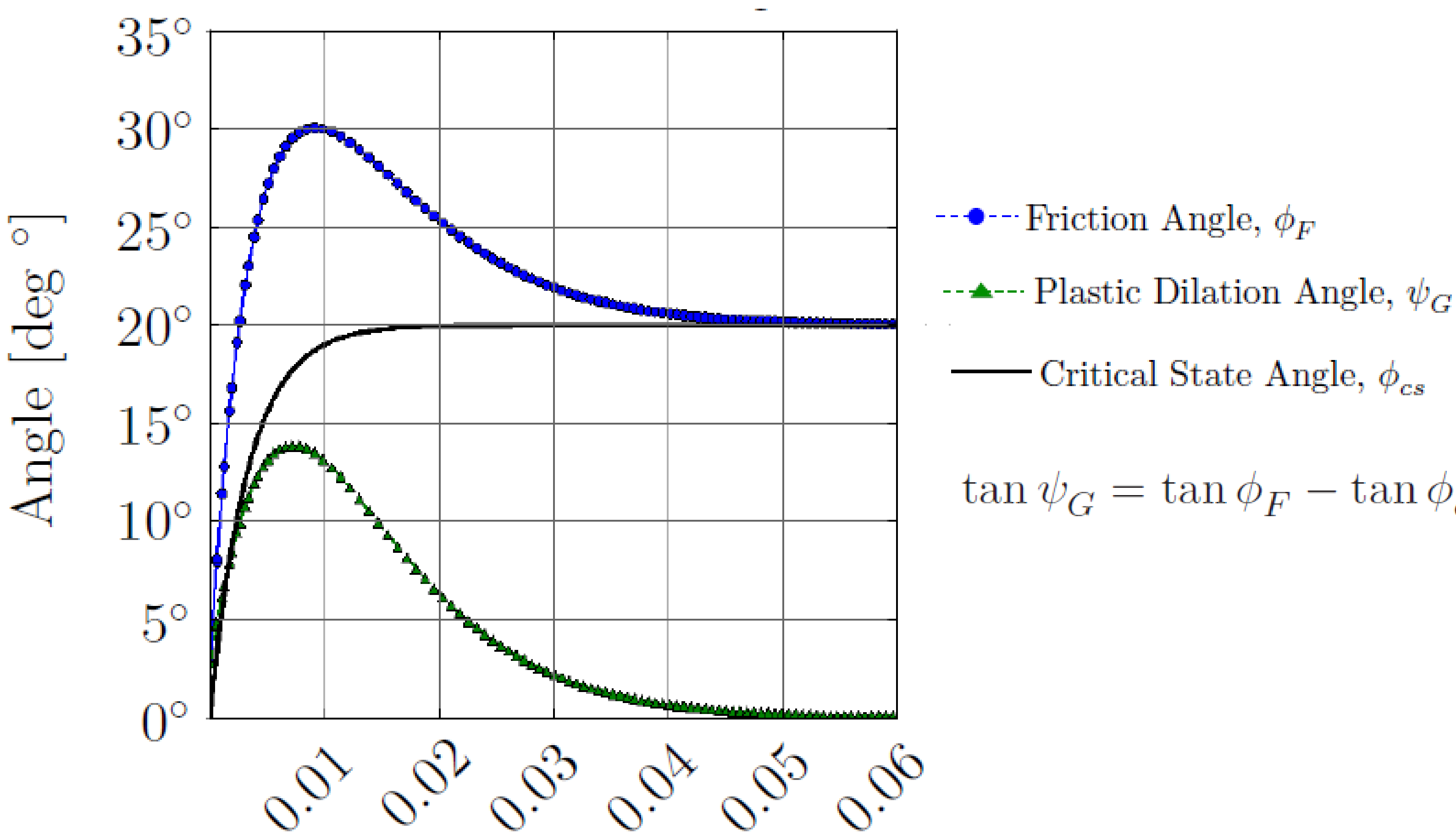
Modeling Granular Materials

- Material Models



Modeling Granular Materials

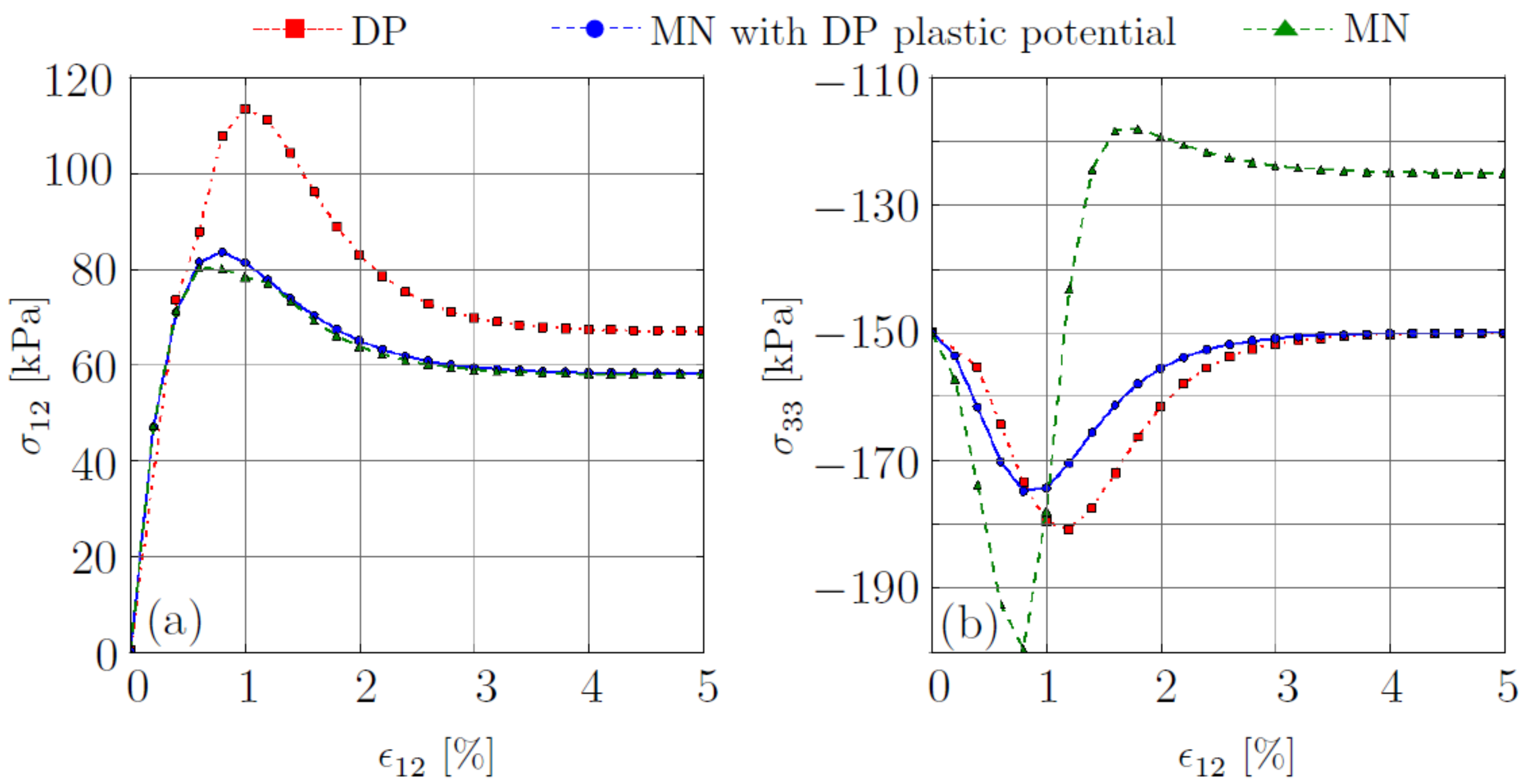
- Material Models



$$\tan\psi_G = \tan\phi_F - \tan\phi_{cs}$$

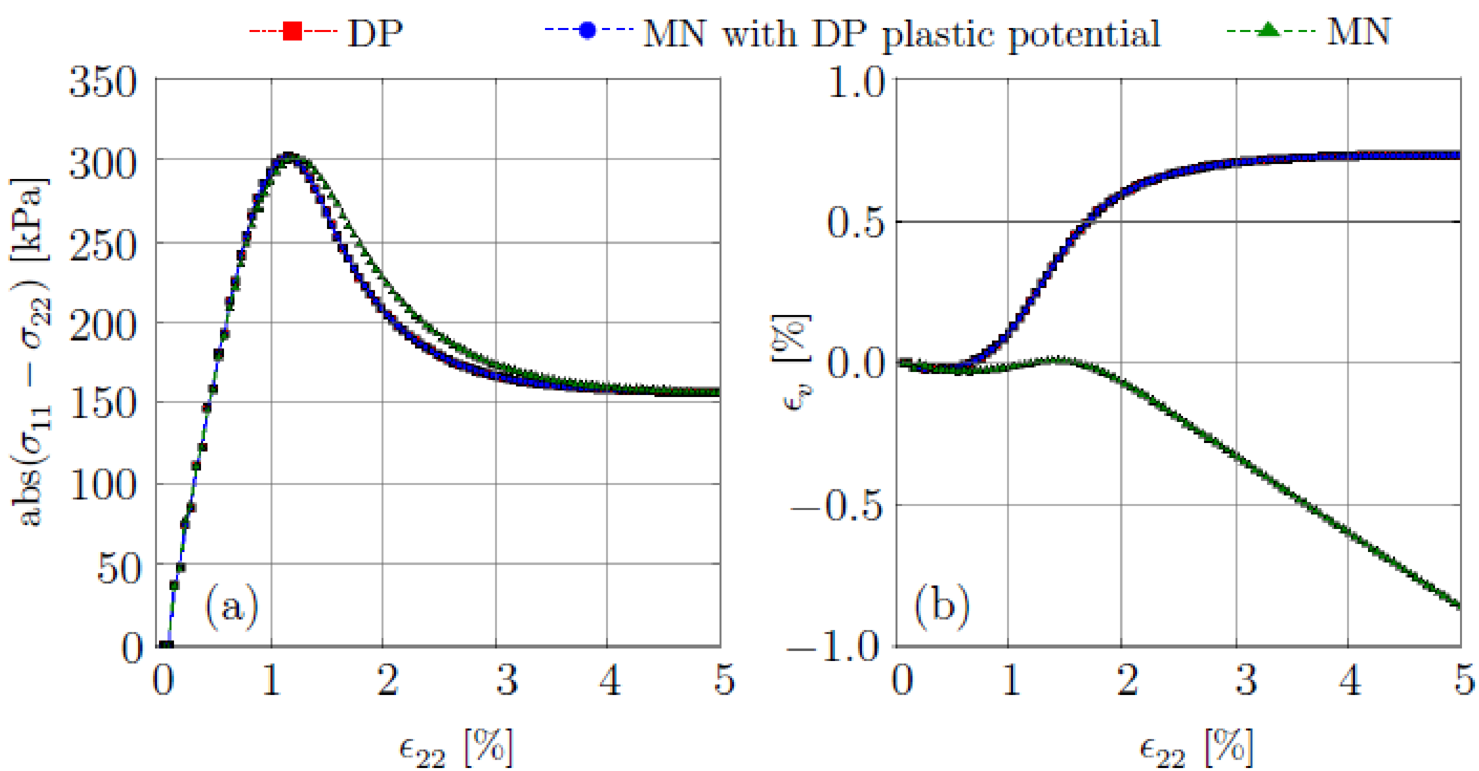
Modeling Granular Materials

- Model Validation – Simple Shear Tests



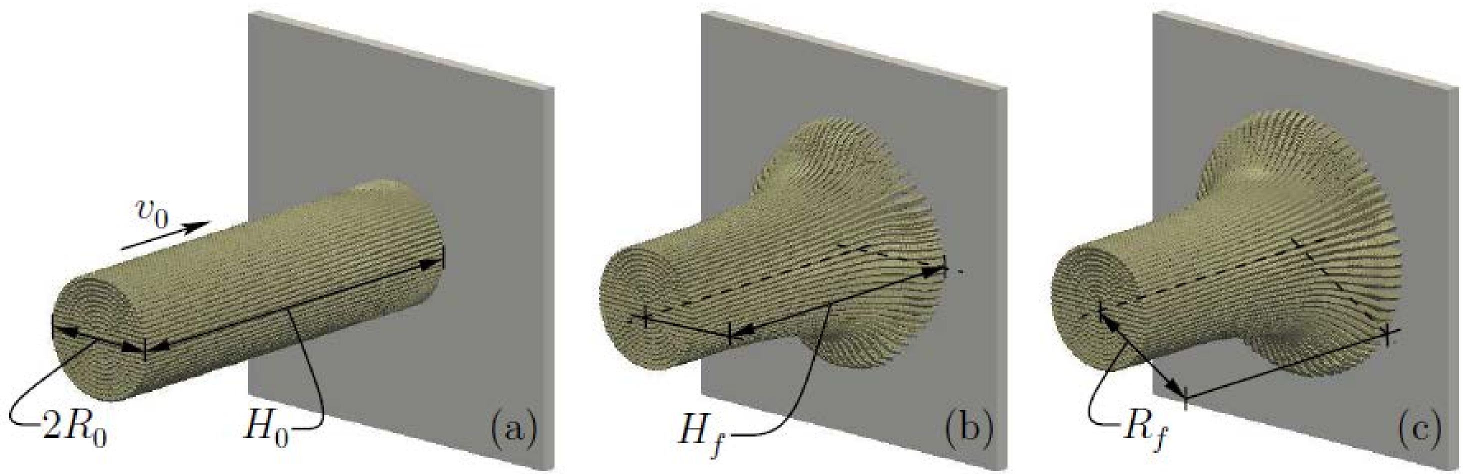
Modeling Granular Materials

- Model Validation – Triaxial Compression



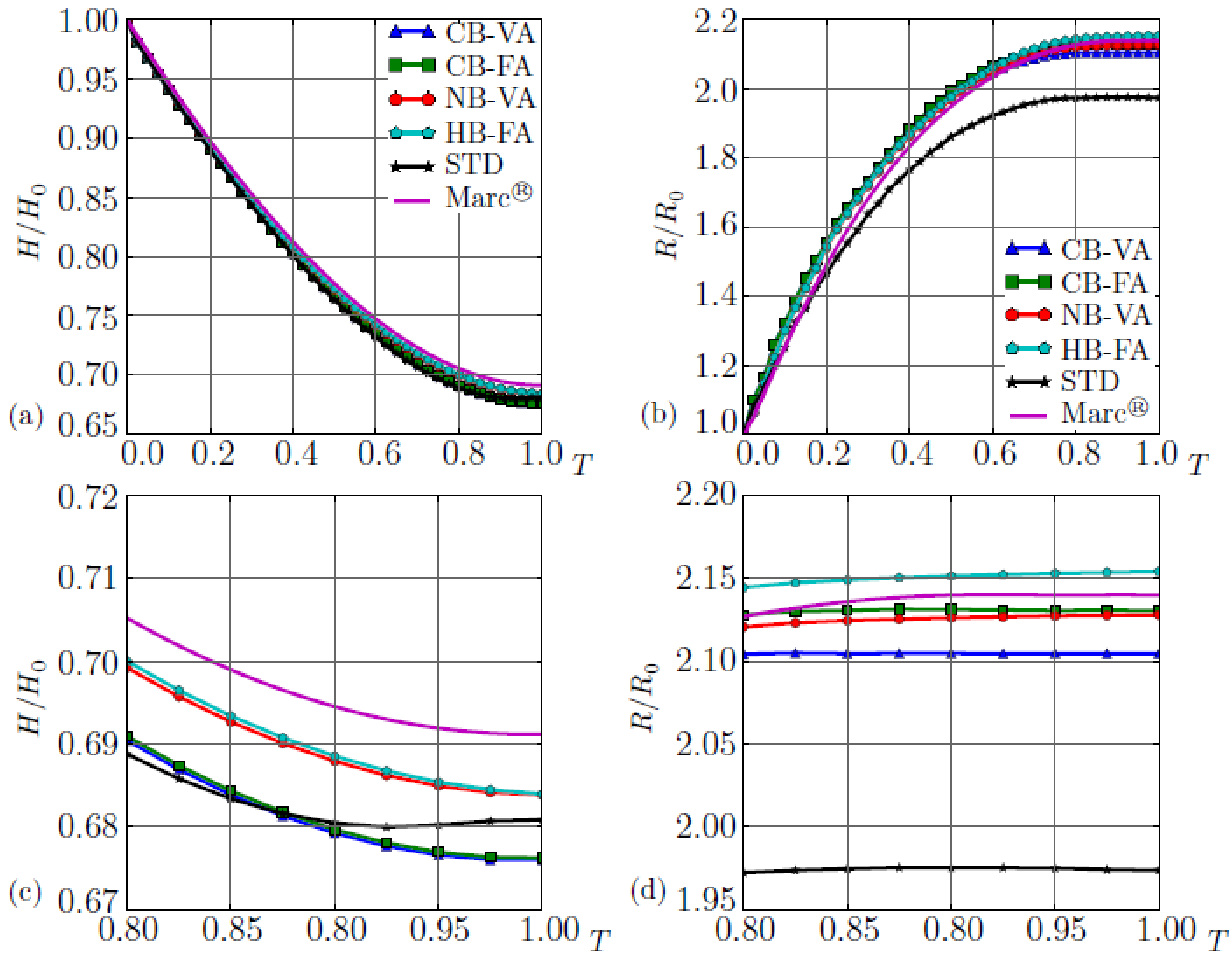
Applications

Applications – Taylor Bar Impact



$$\bar{H}(t) = \frac{H(t)}{H_0}, \qquad \bar{R}(t) = \frac{R(t)}{R_0}, \qquad \text{and} \qquad T = \frac{t}{t_f},$$

Applications – Taylor Bar Impact



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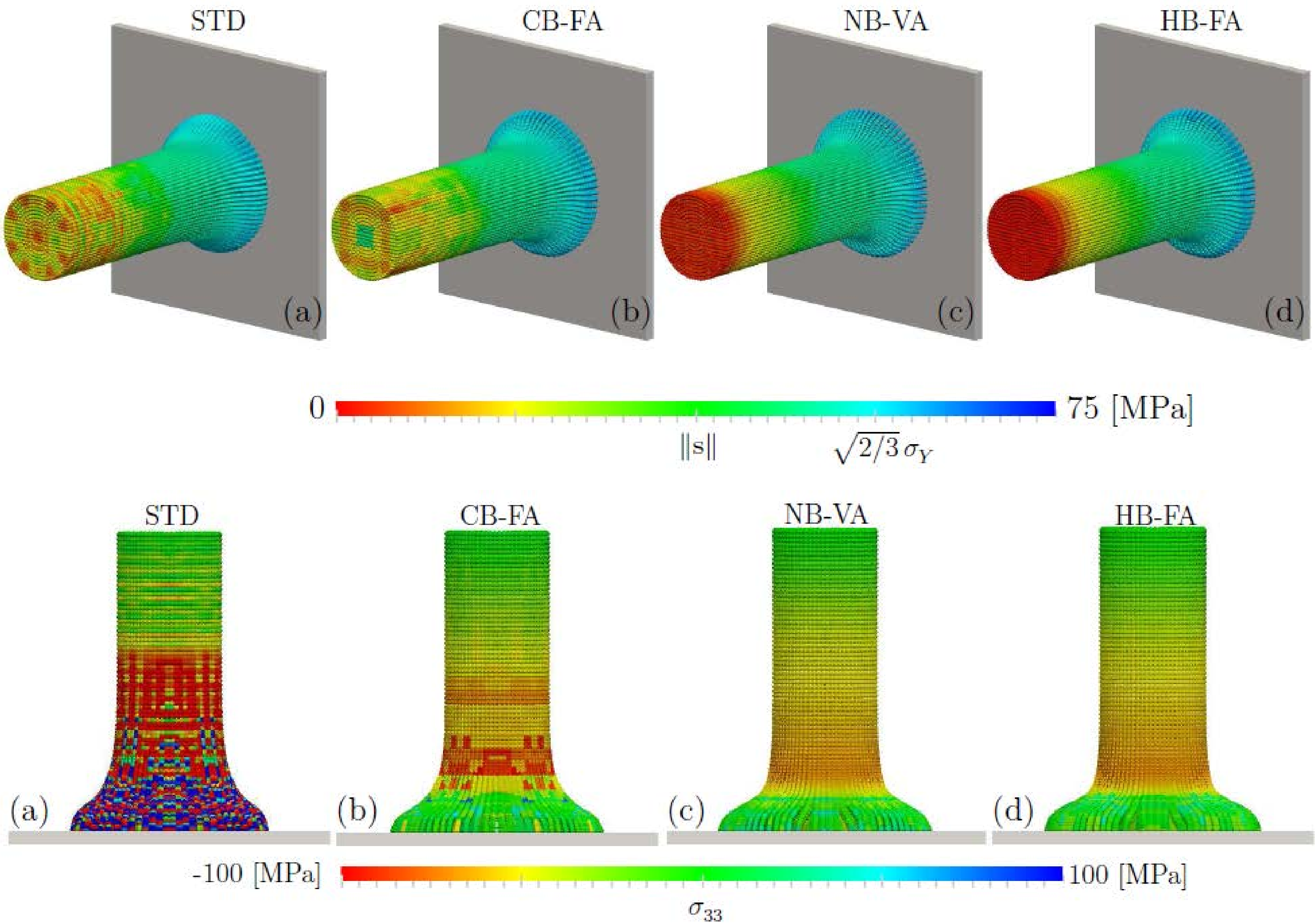

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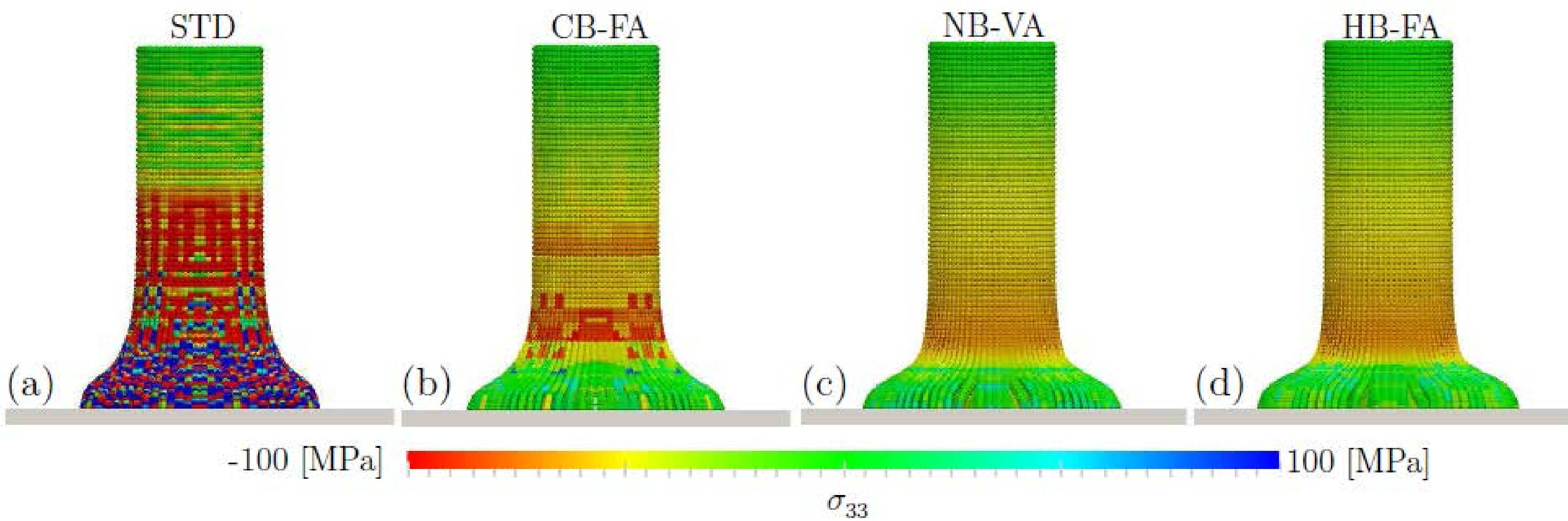

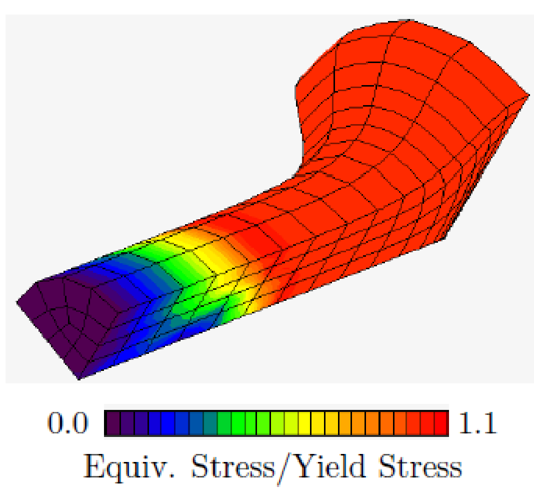

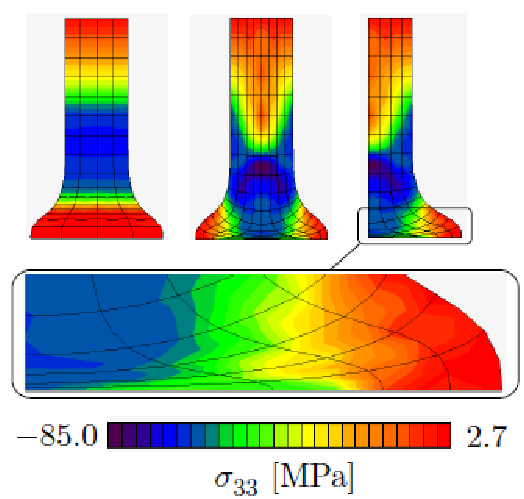

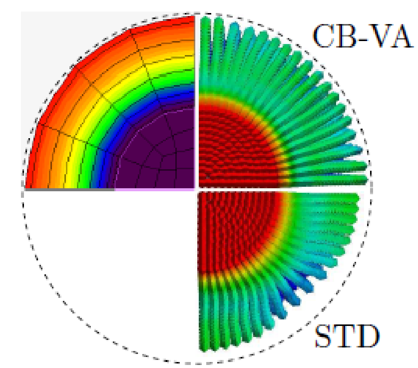

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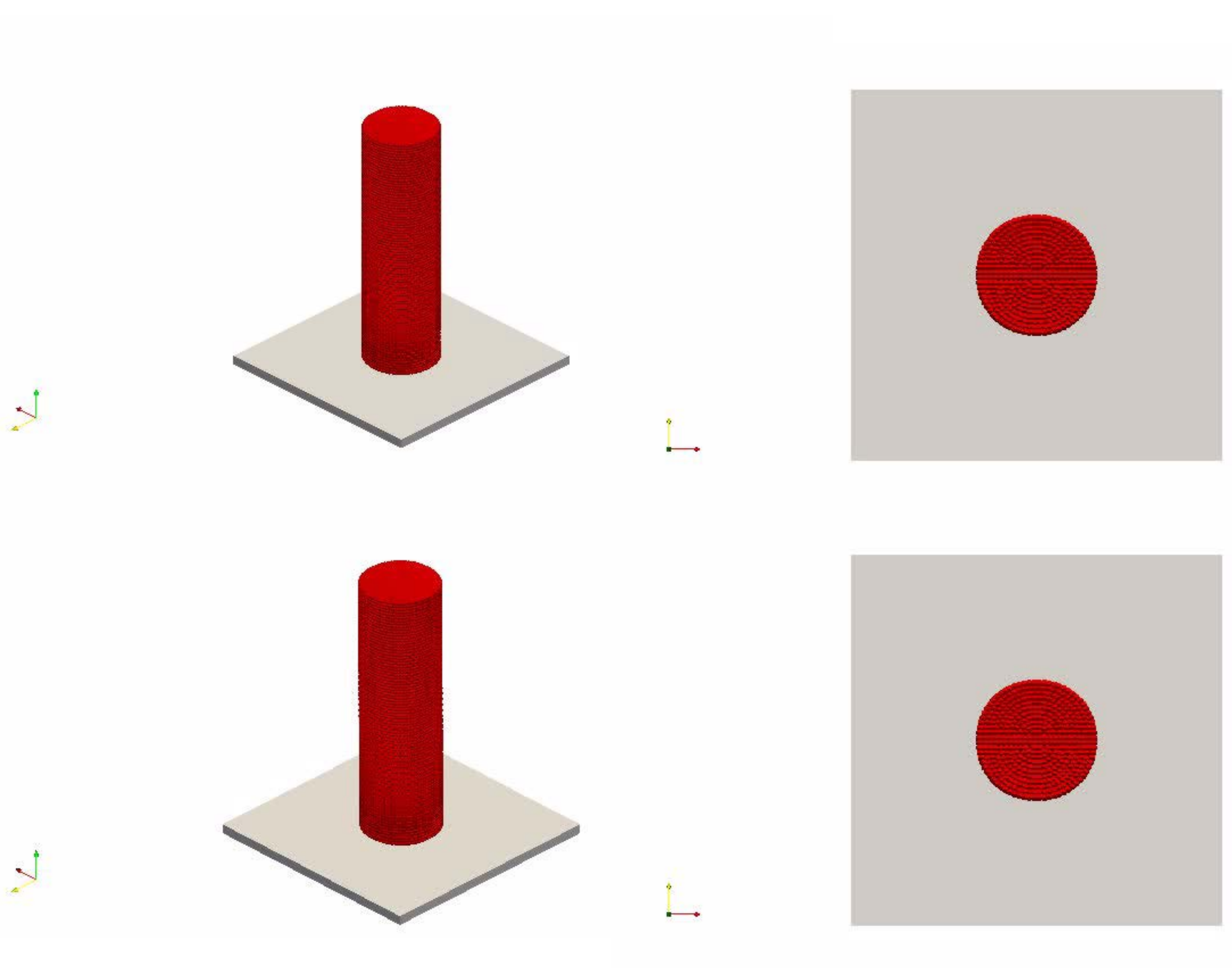
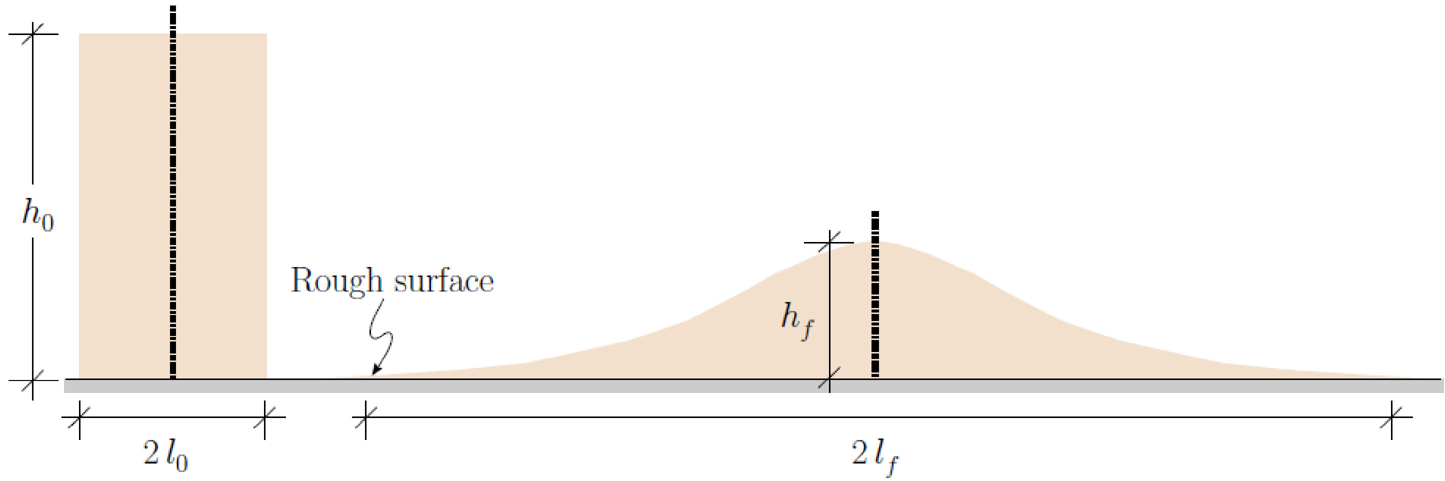
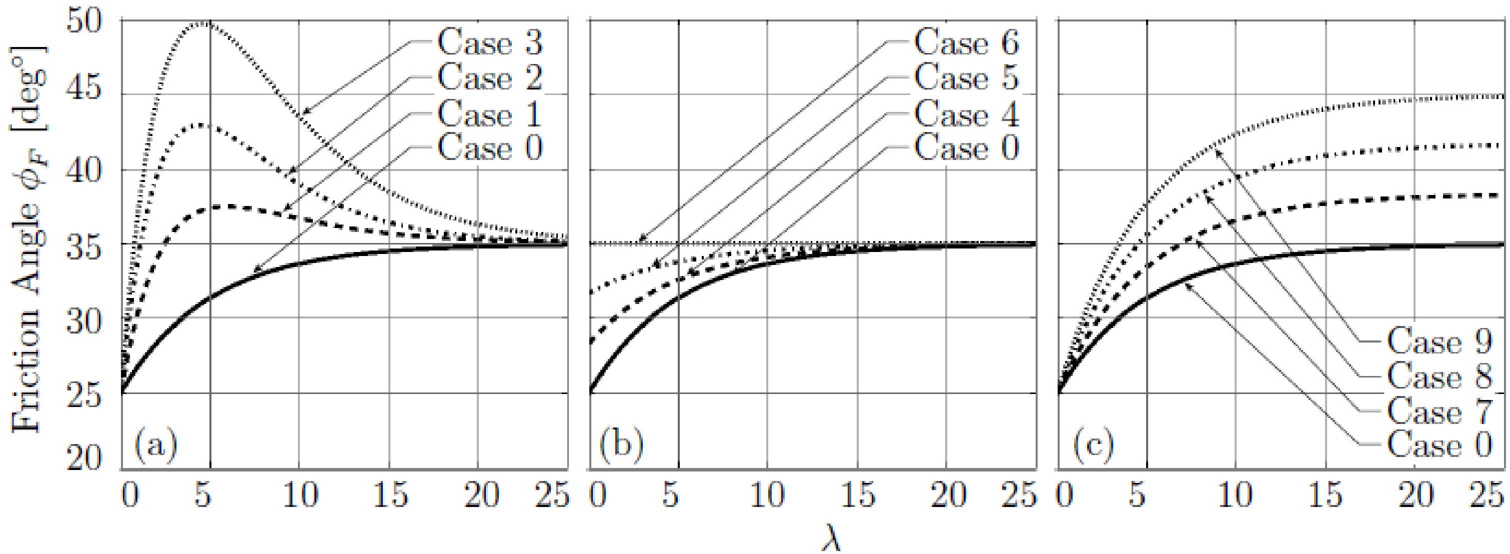


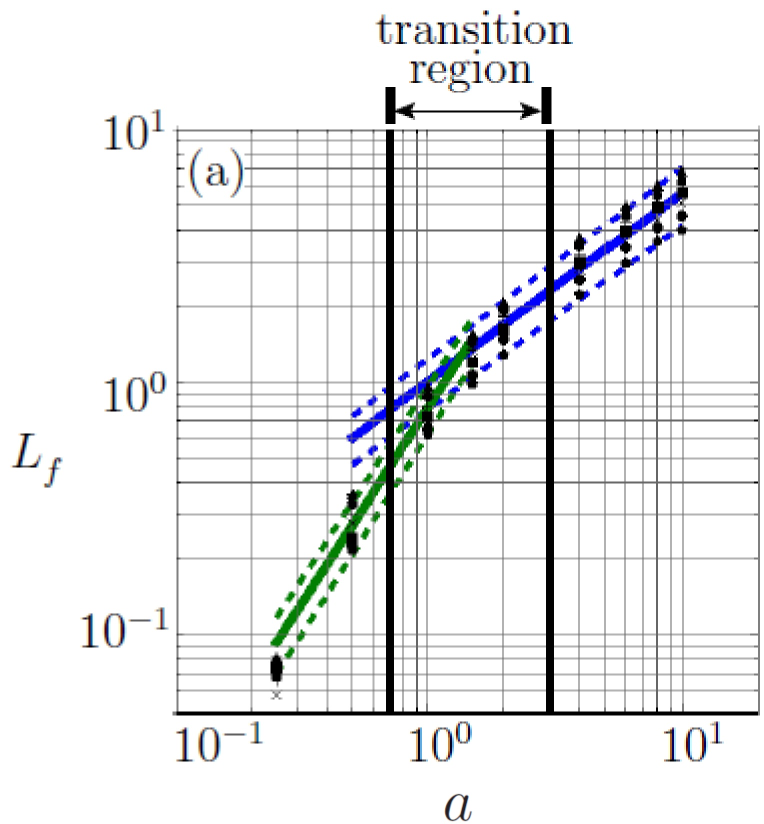
Applications – Planar Sand Column Collapse

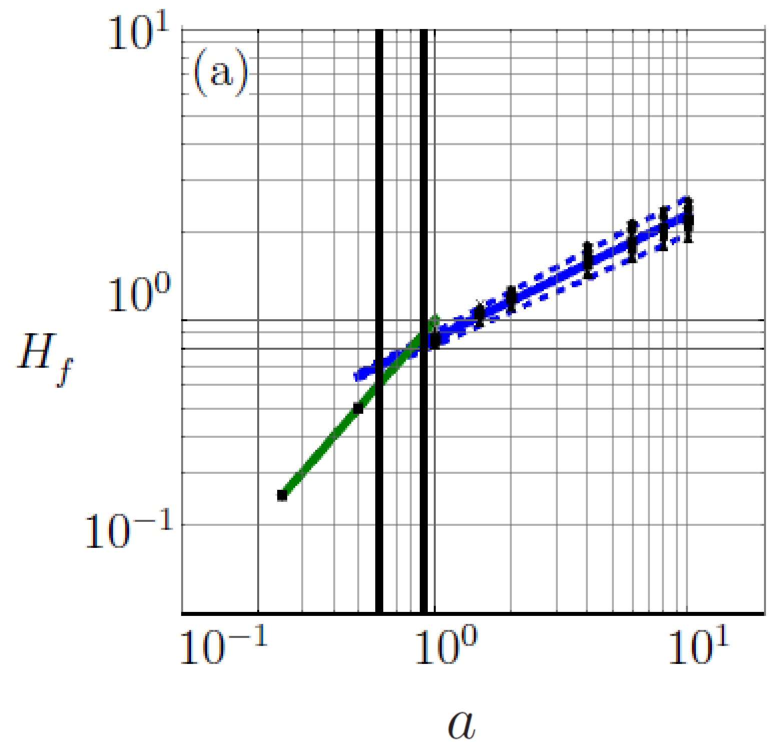




Applications – Planar Sand Column Collapse

$$a = \frac{h_0}{l_0} \qquad H(t) = \frac{h(t)}{l_0}, \qquad L(t) = \frac{l(t) - l_0}{l_0}, \qquad \text{and} \qquad T = \frac{t}{t_f}$$





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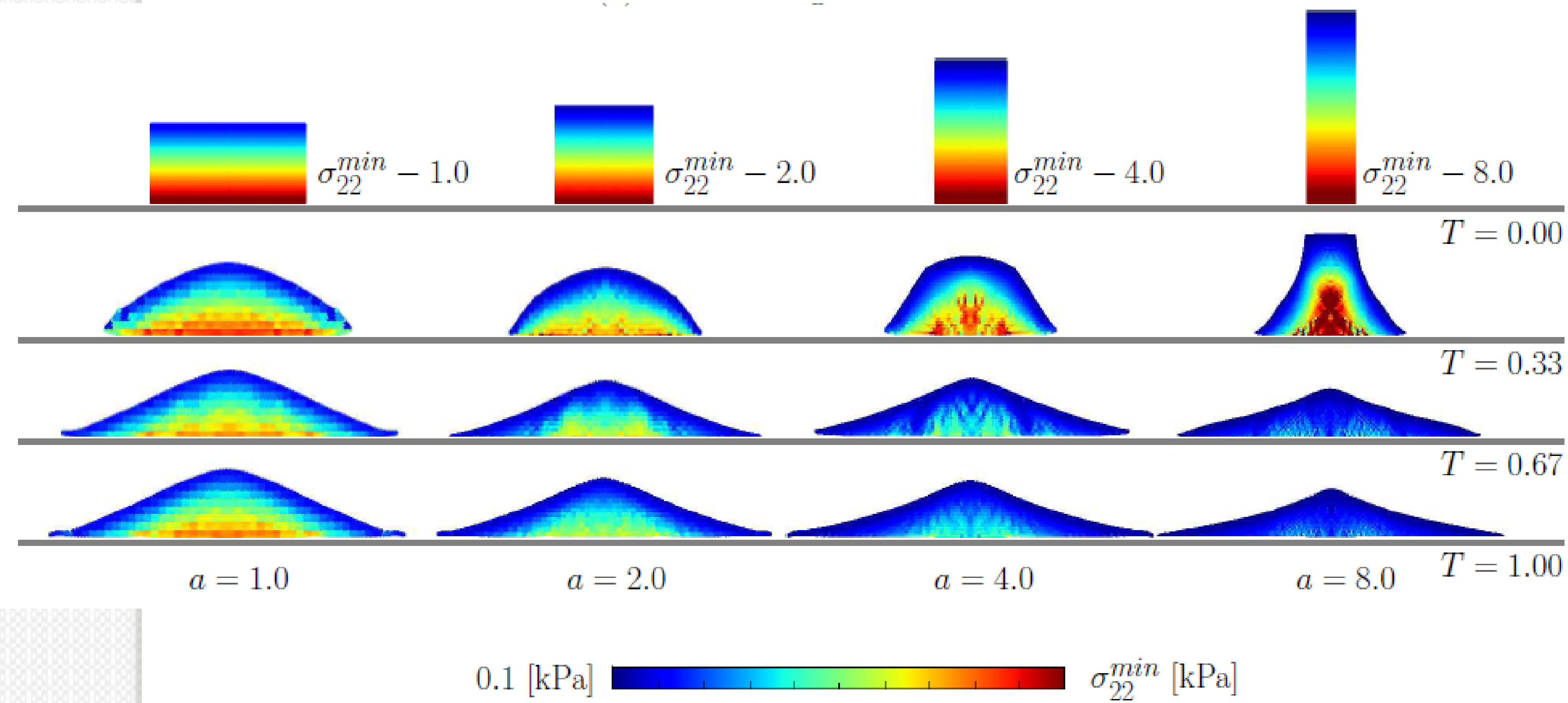

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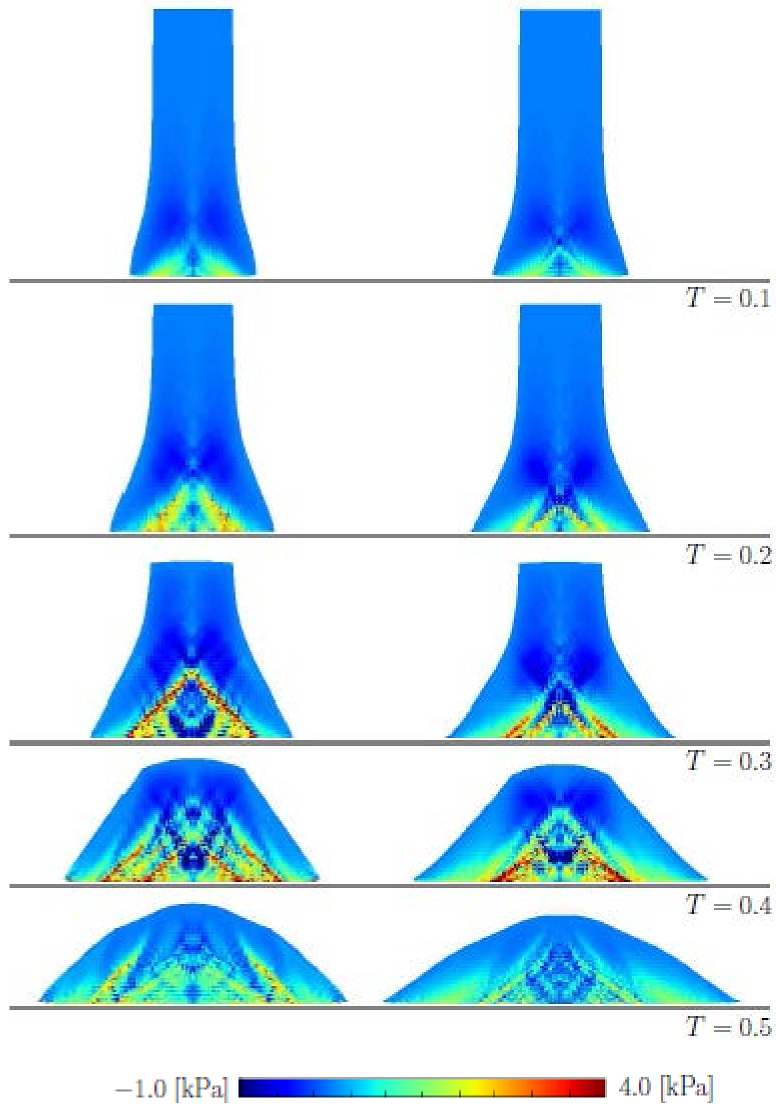

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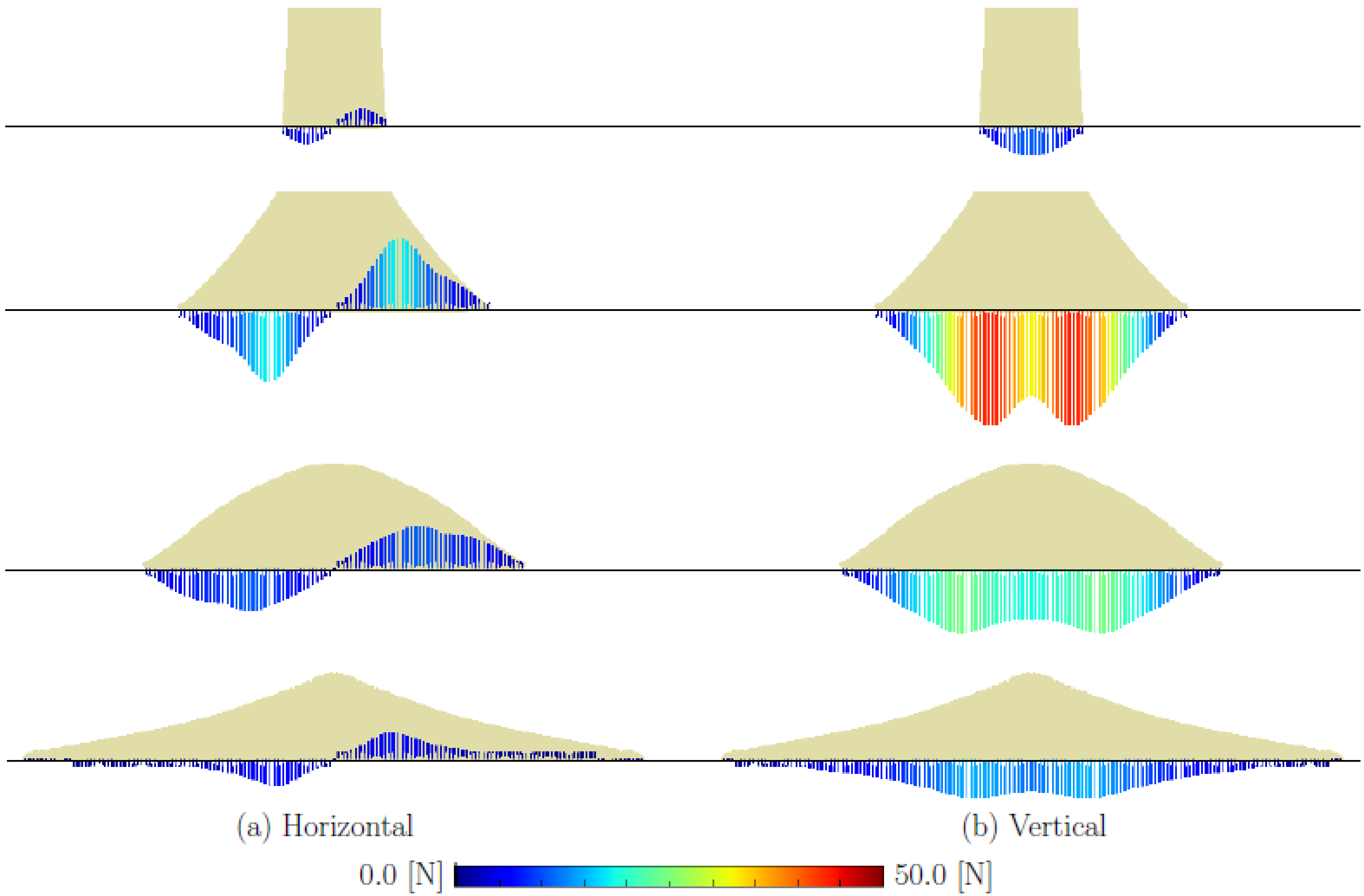
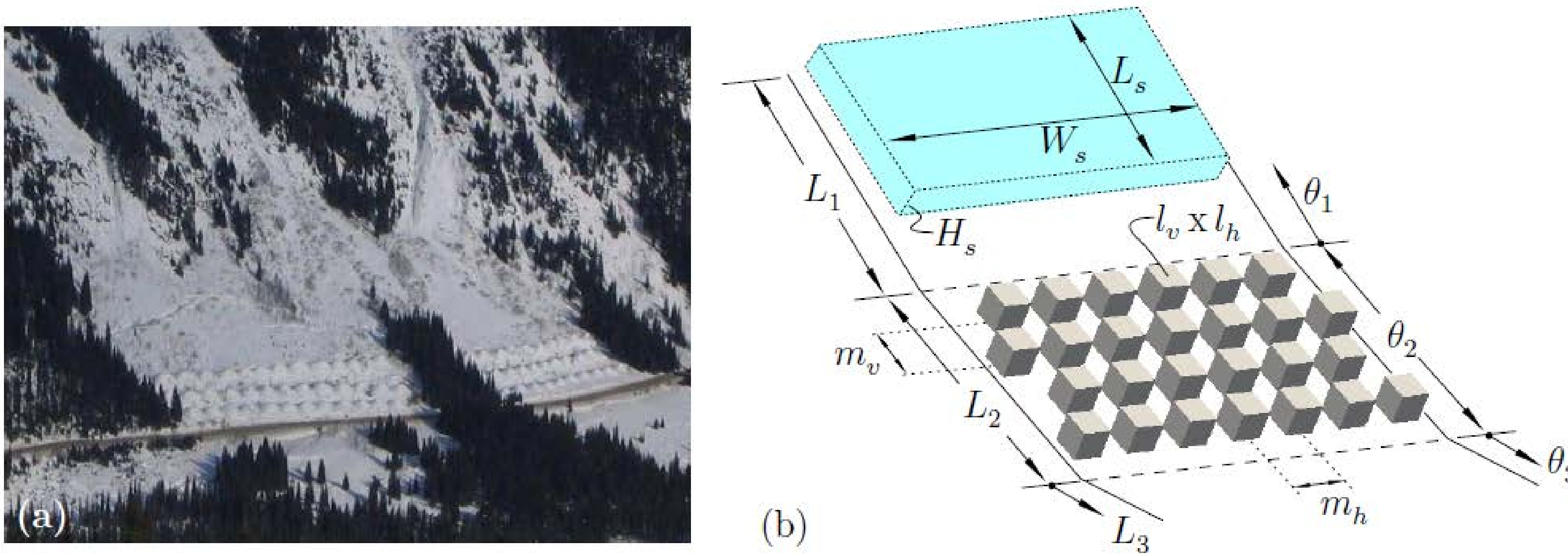


(a) Horizontal (b) Vertical

Applications – Avalanche Control



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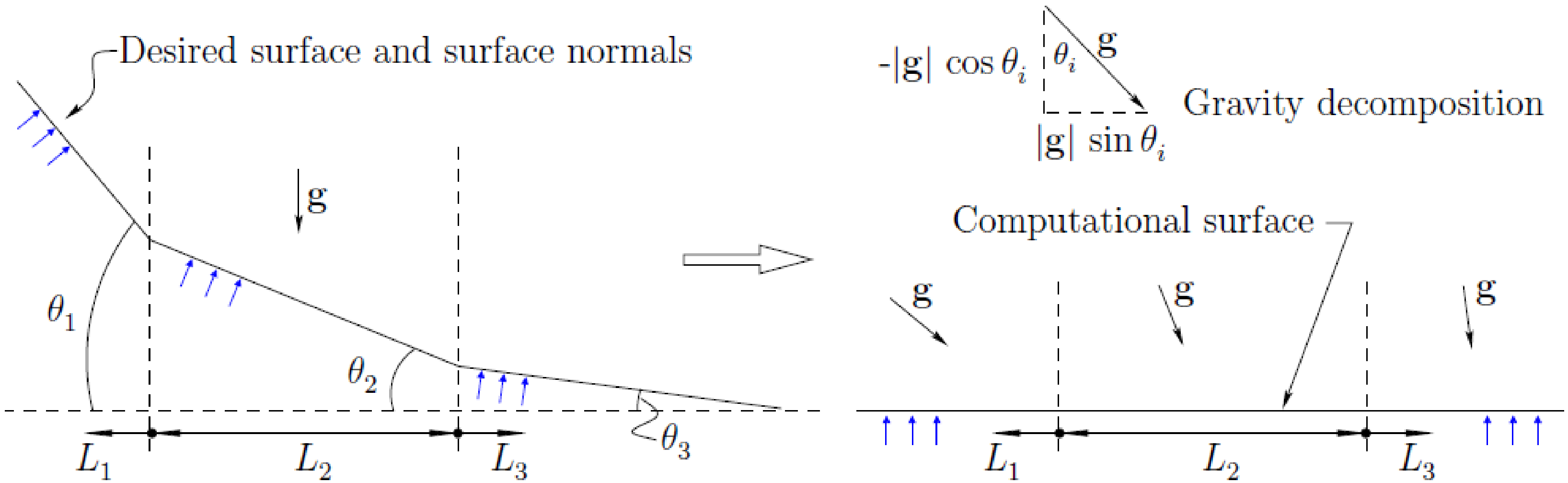

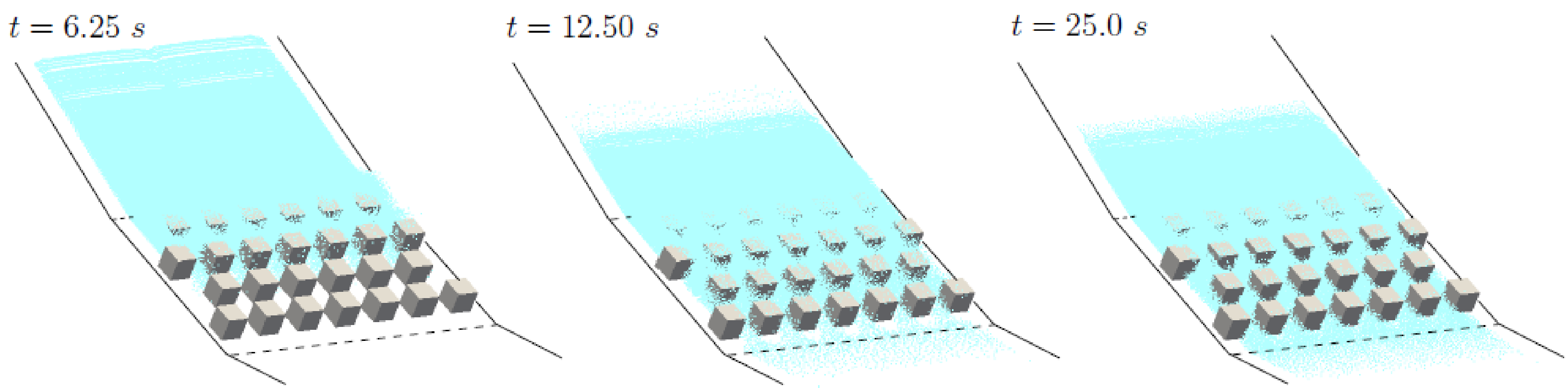

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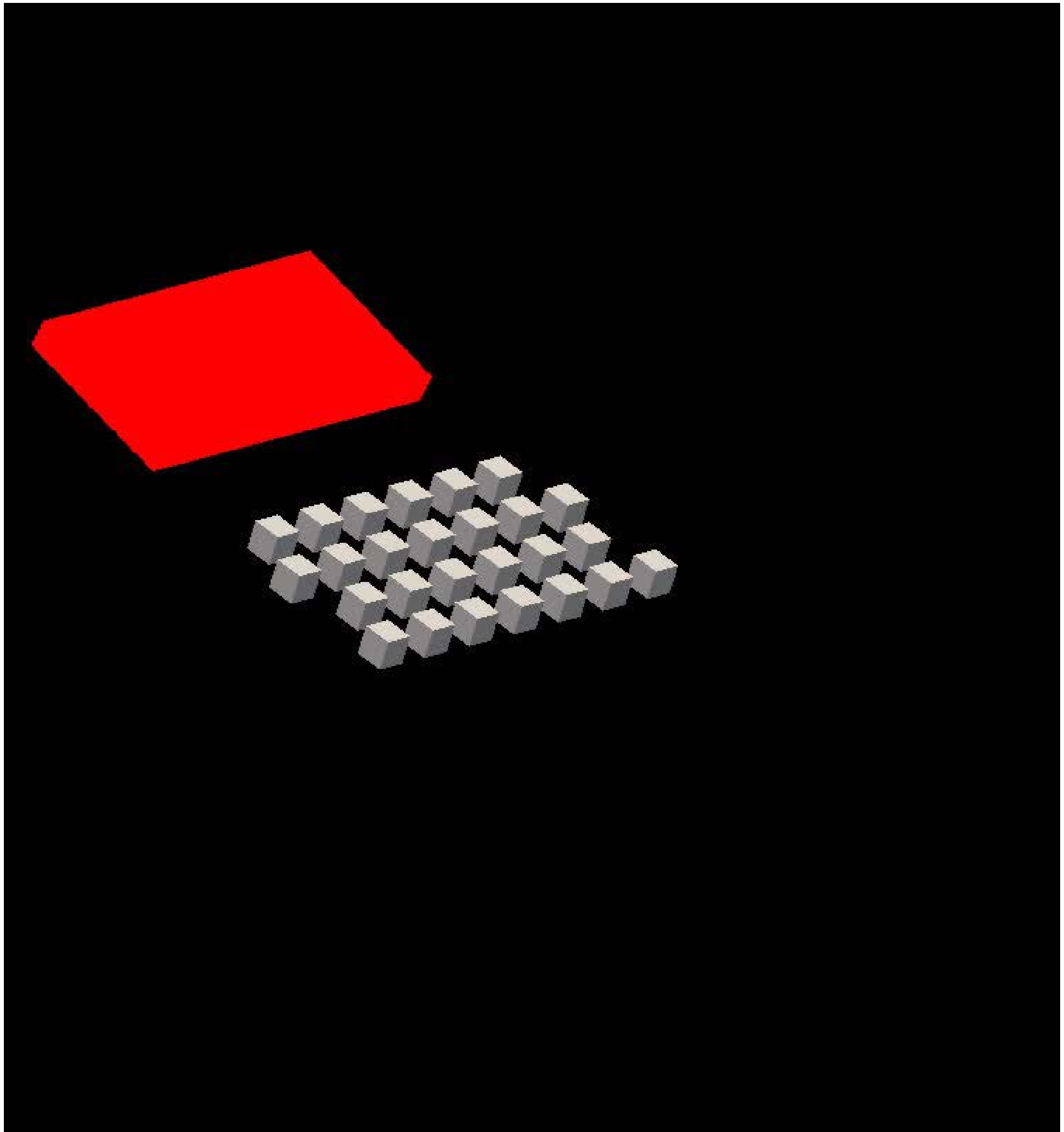

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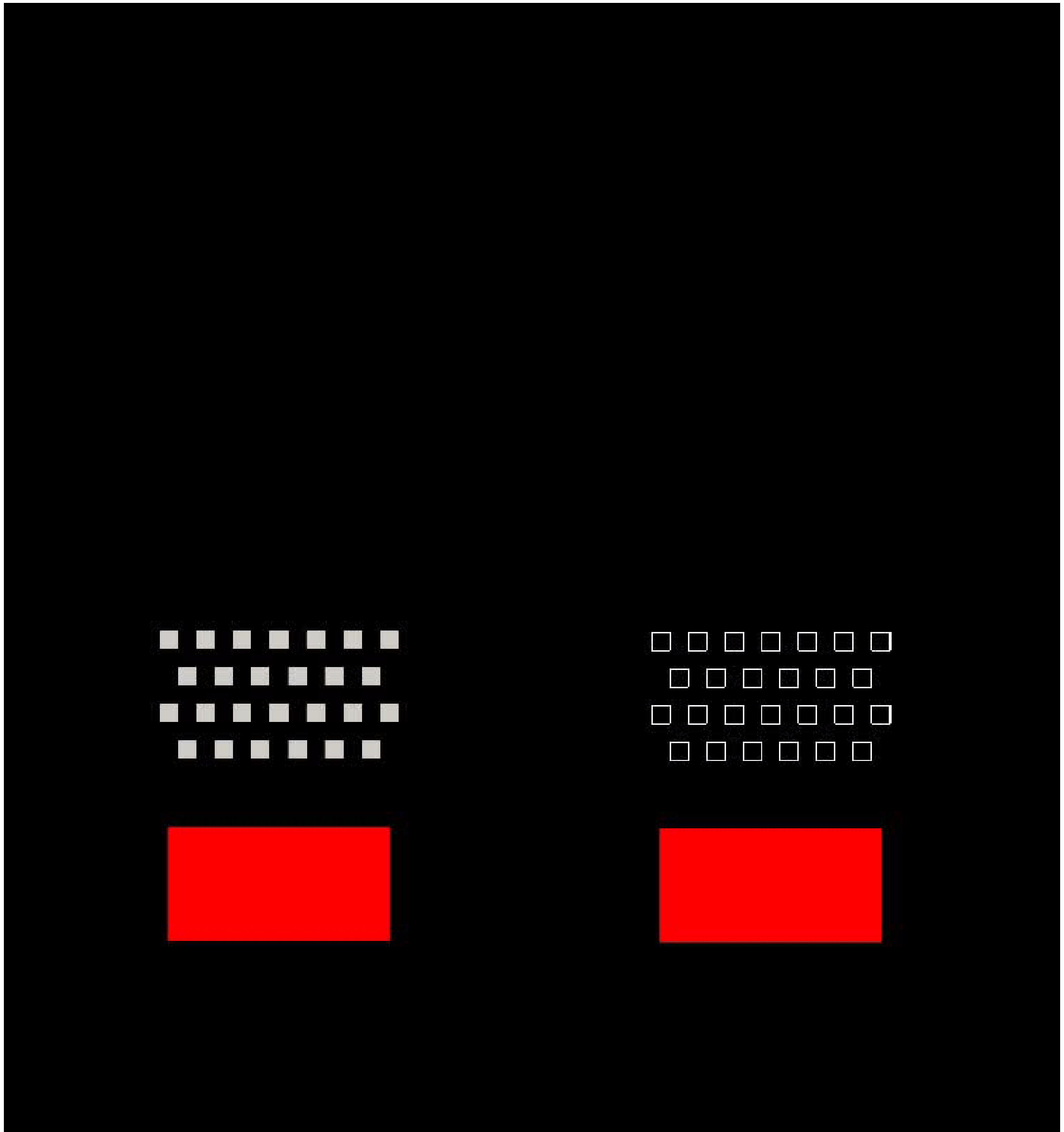

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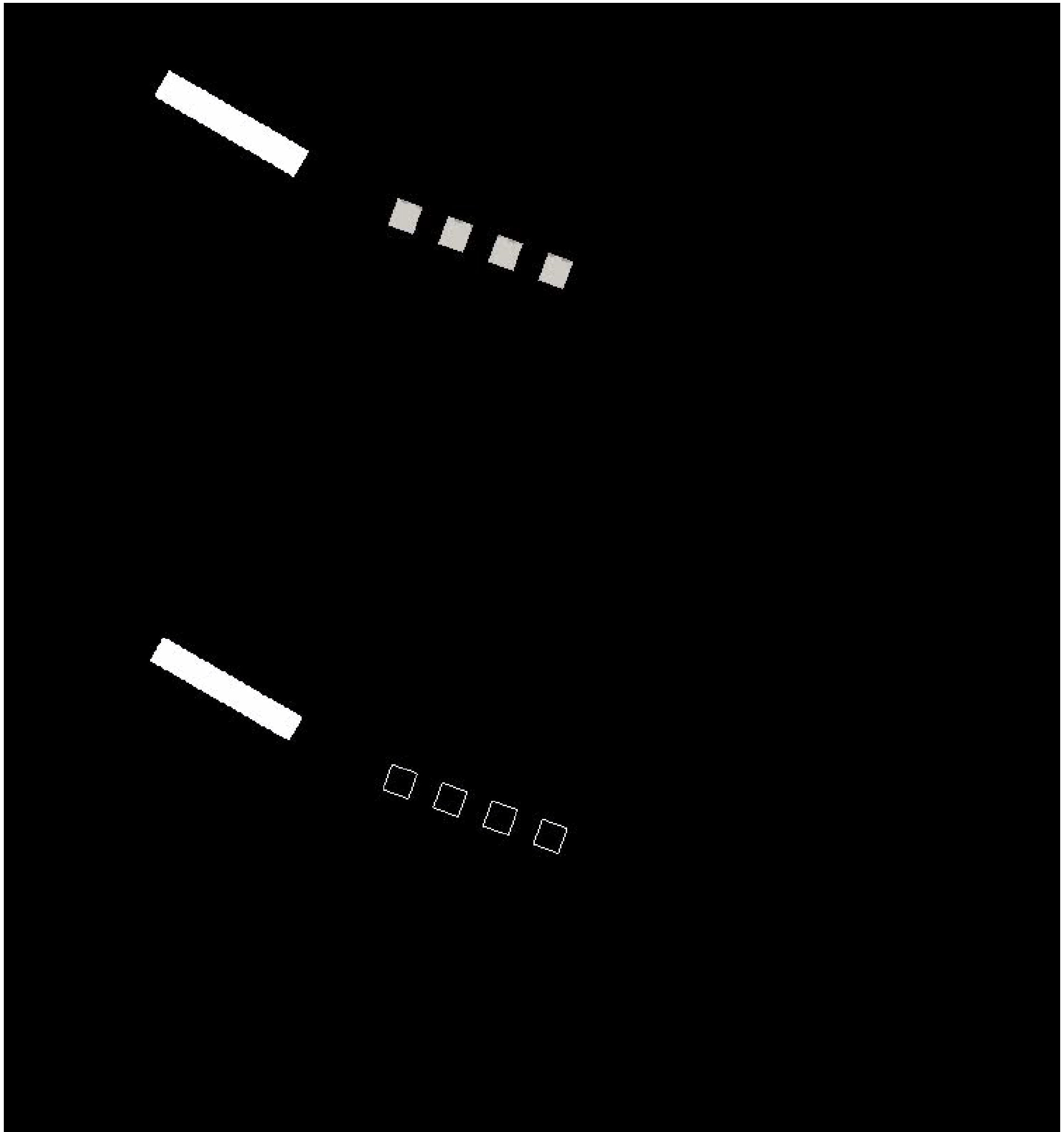

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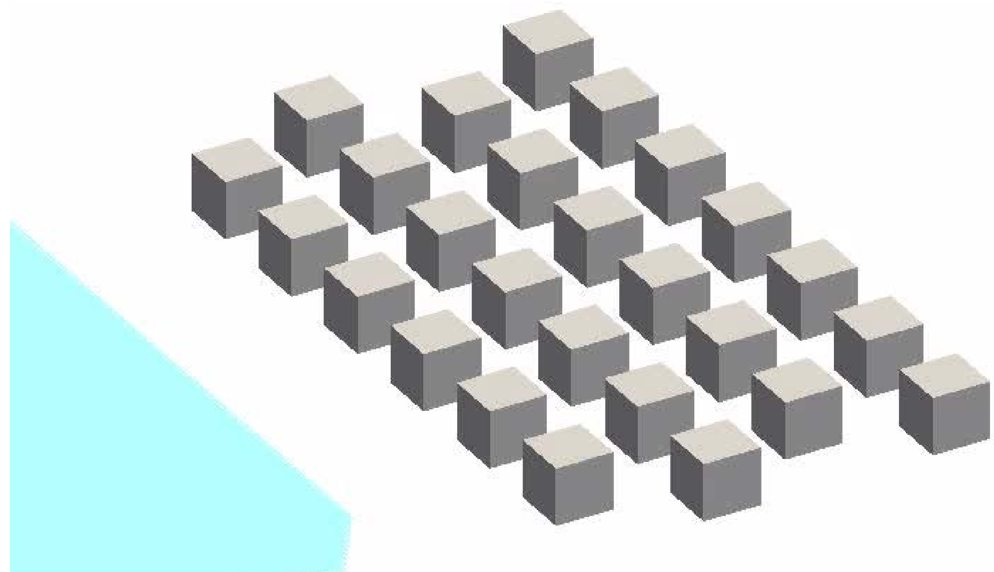
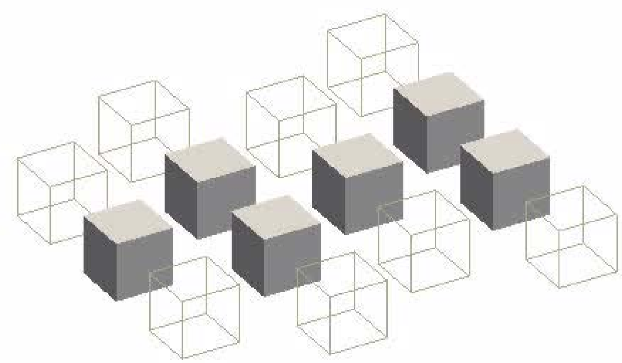
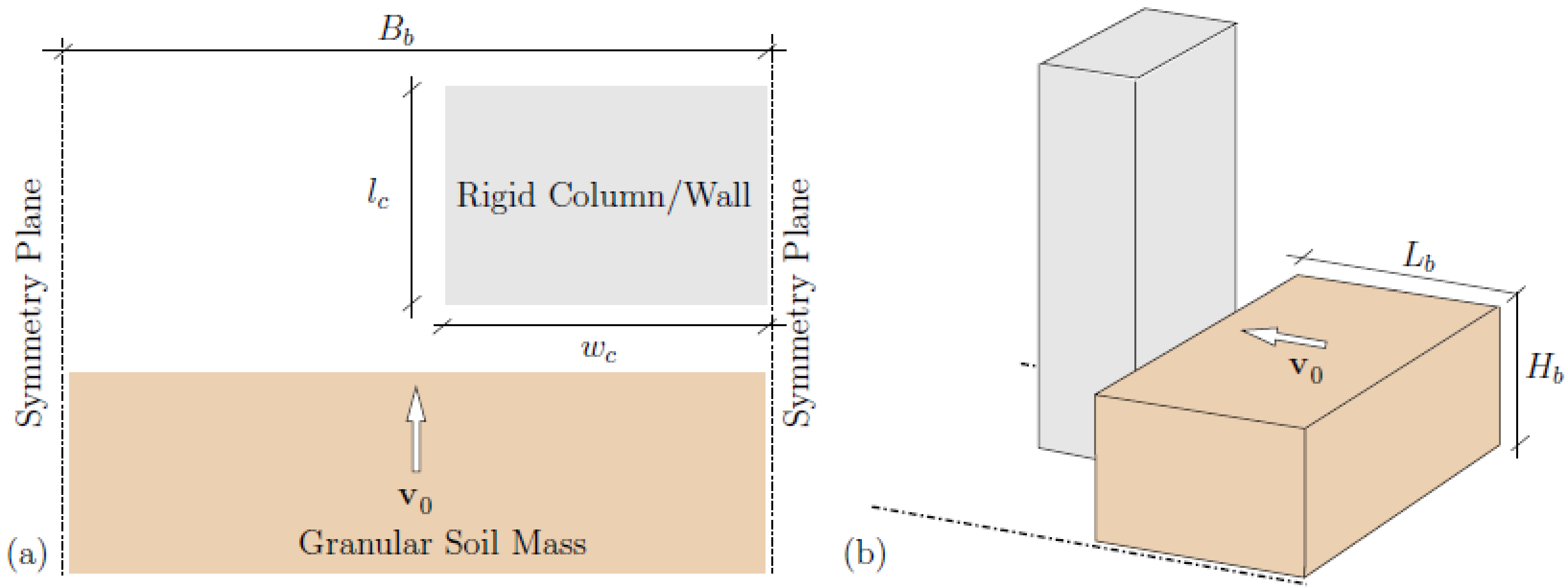




Applications – Parametric Force Analysis



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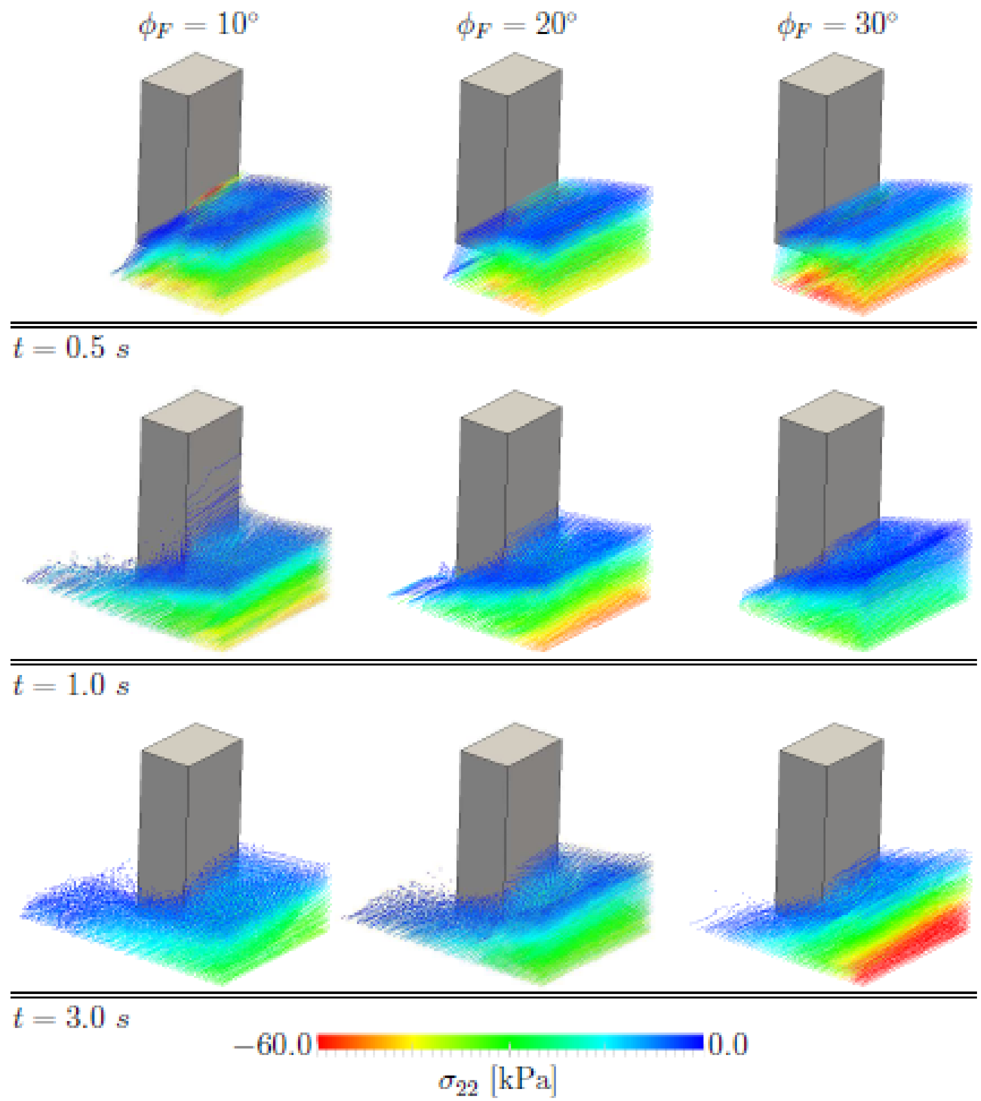

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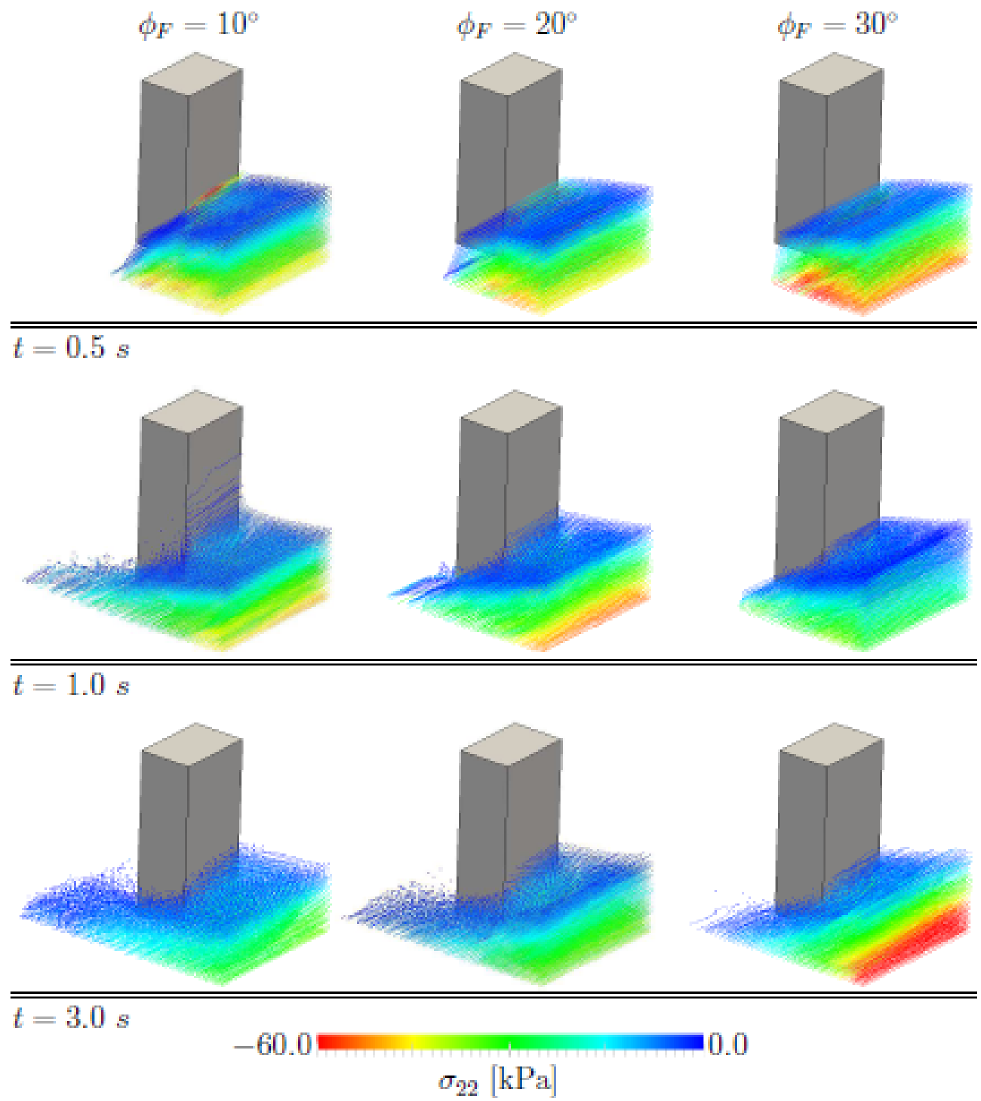

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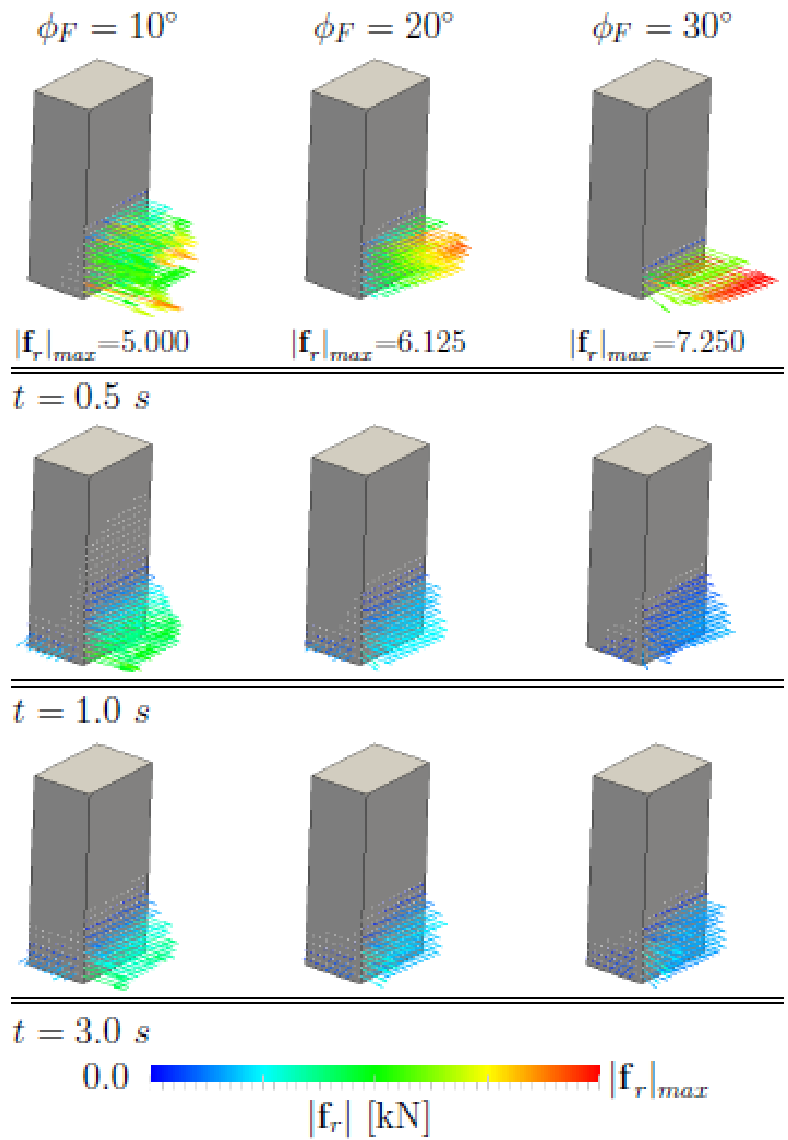

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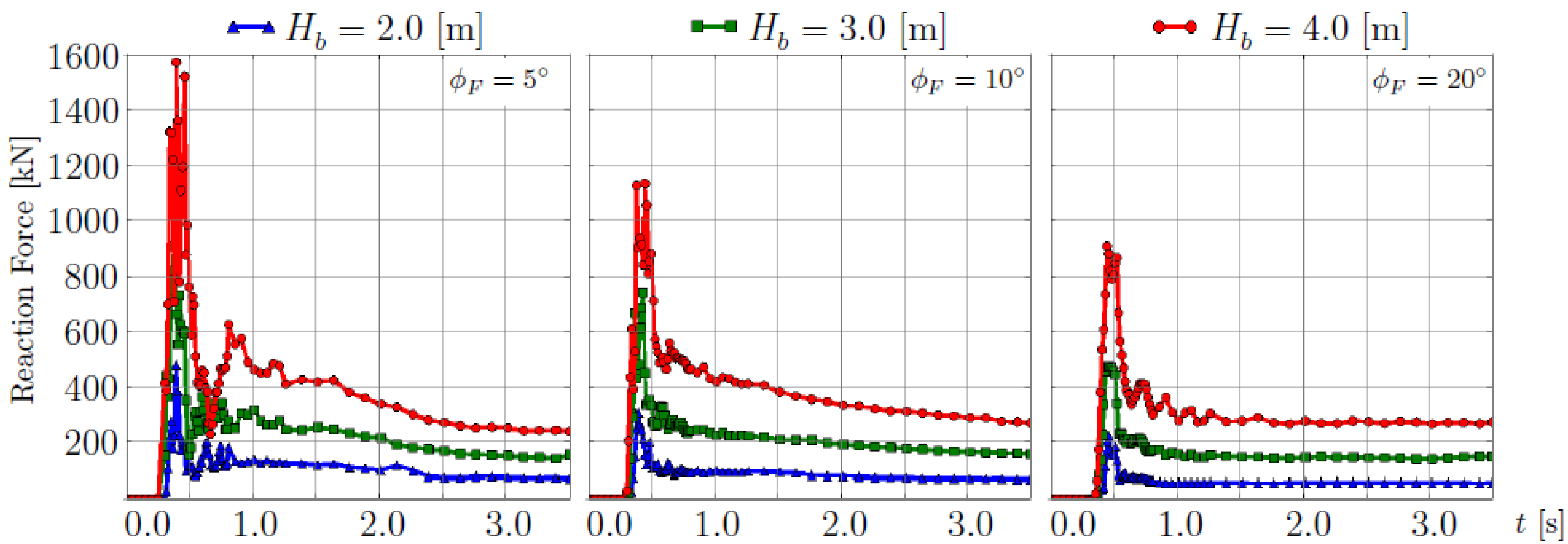

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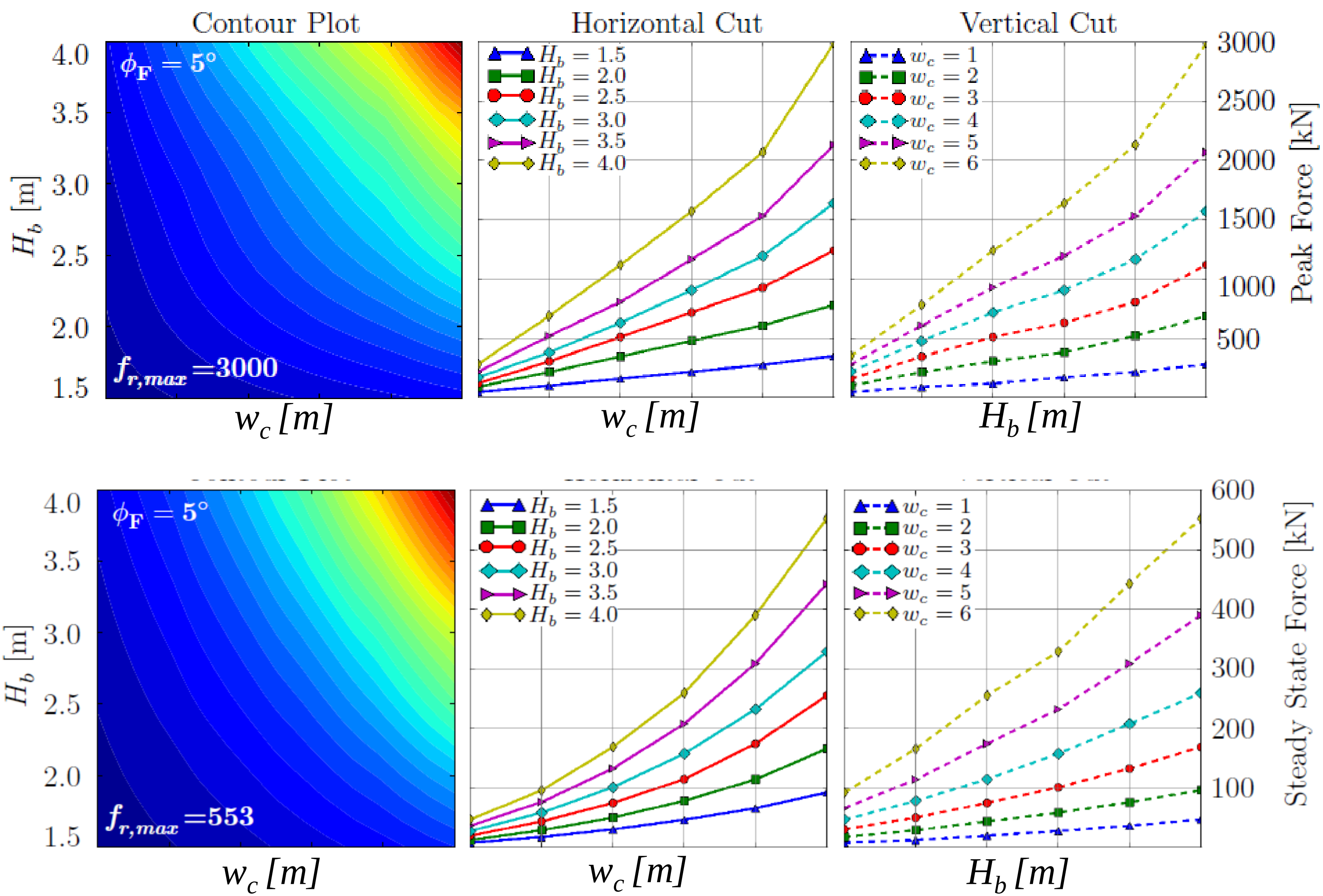
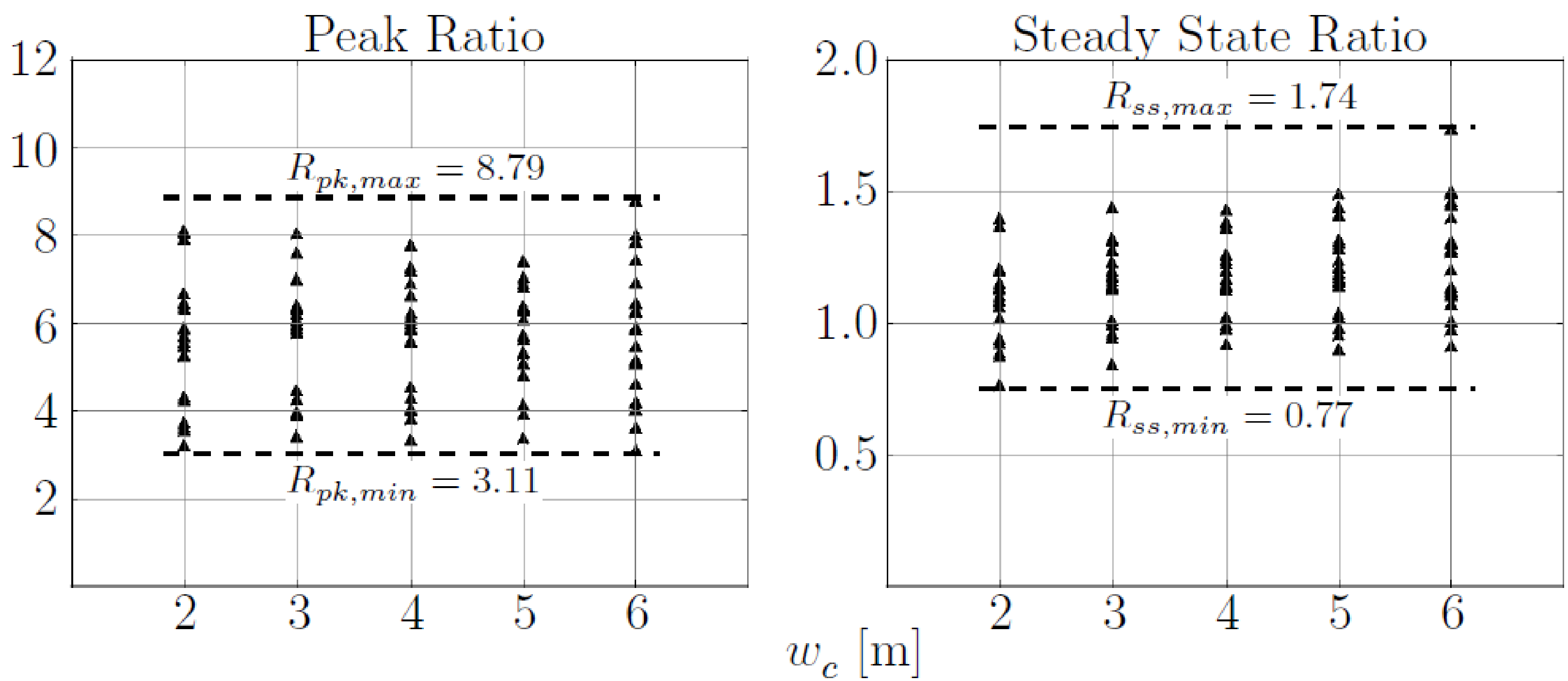


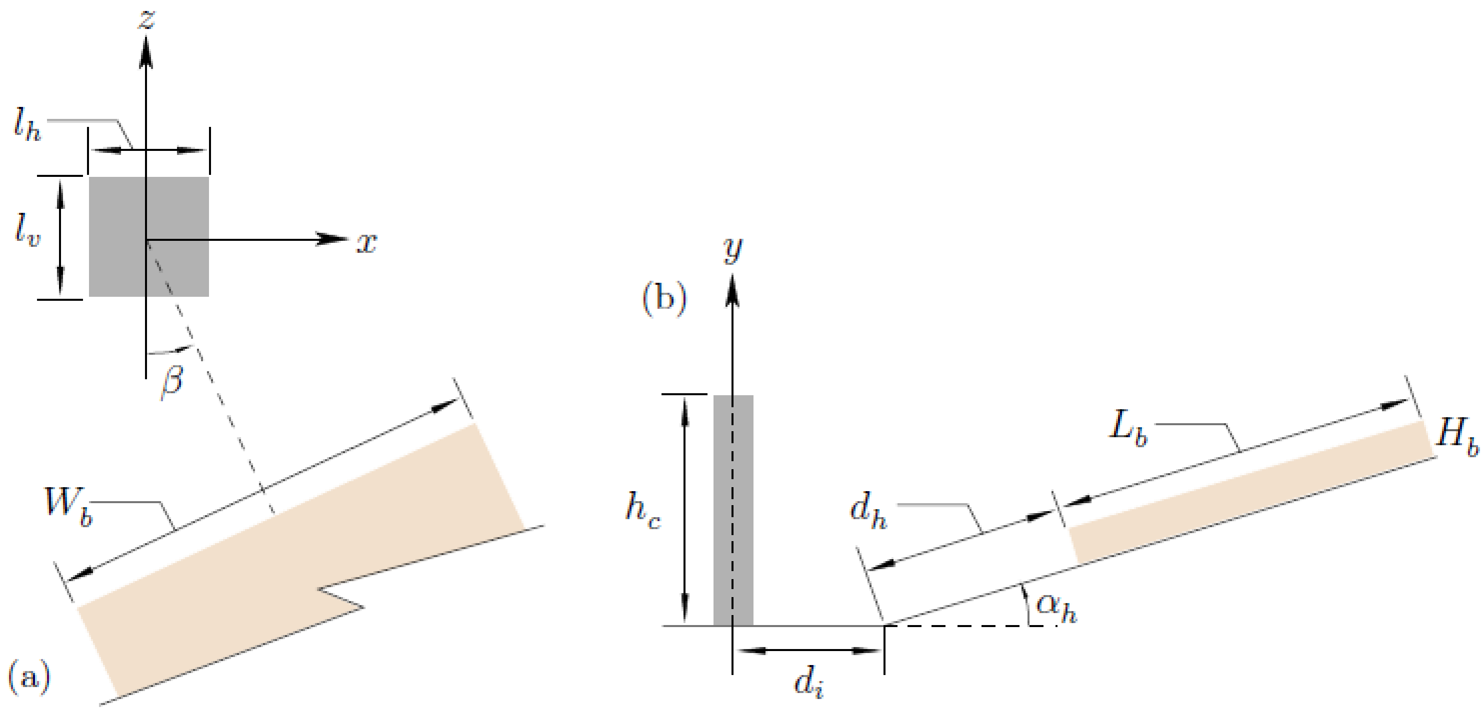
Applications – Parametric Force Analysis

$$b_{st} = \frac{1}{2} K_0 \, \rho_0 \, |\mathbf{g}| \, H_{act}^2 \qquad f_{st} = b_{st} \, w_c = \frac{1}{2} K_0 \, \rho_0 \, |\mathbf{g}| \, H_{act}^2 \, w_c$$

$$R_{pk} = \frac{f_{pk}}{f_{st}} \quad \text{and} \quad R_{ss} = \frac{f_{ss}}{f_{st}}$$



Applications – Effects of Approach Angle



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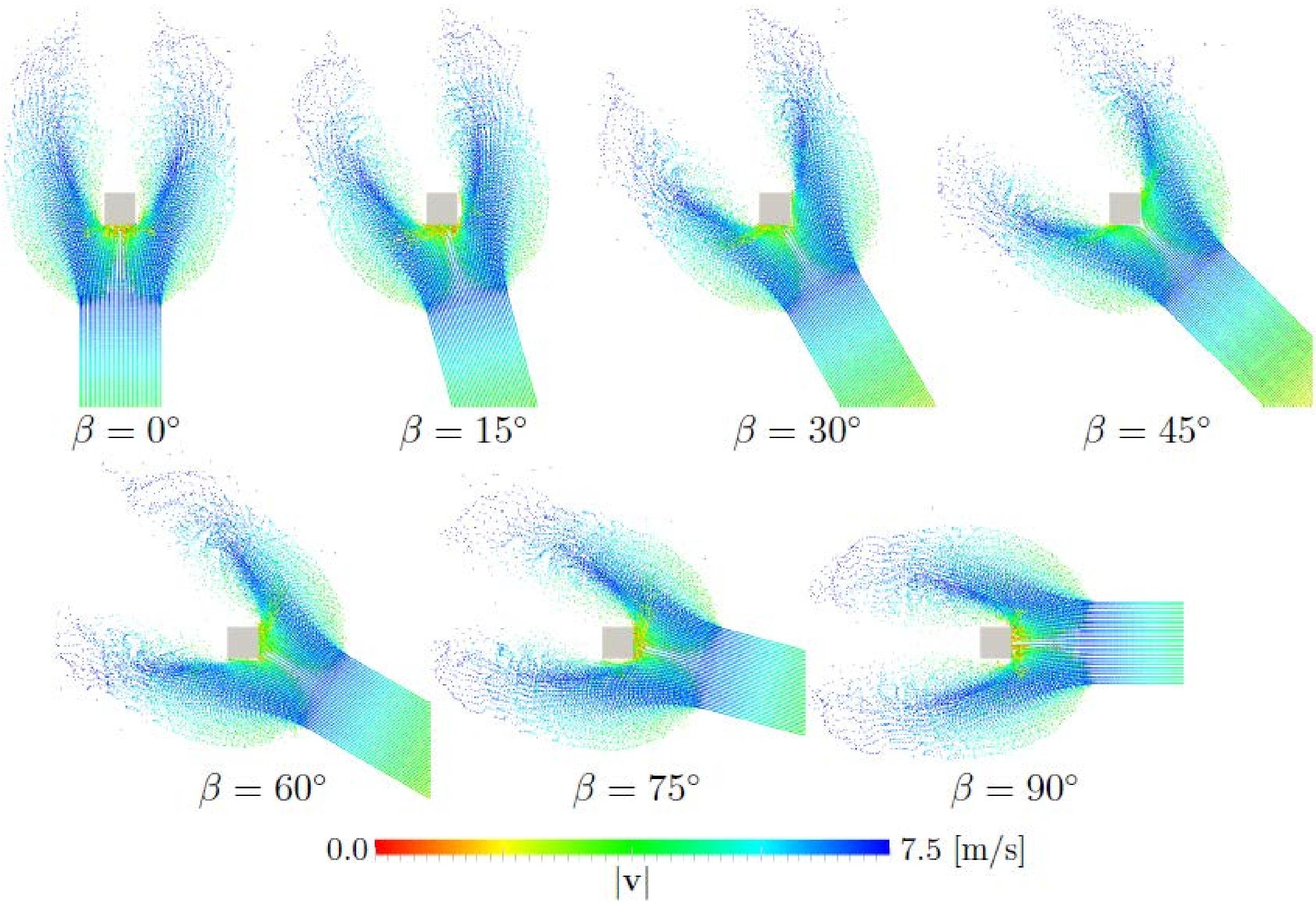

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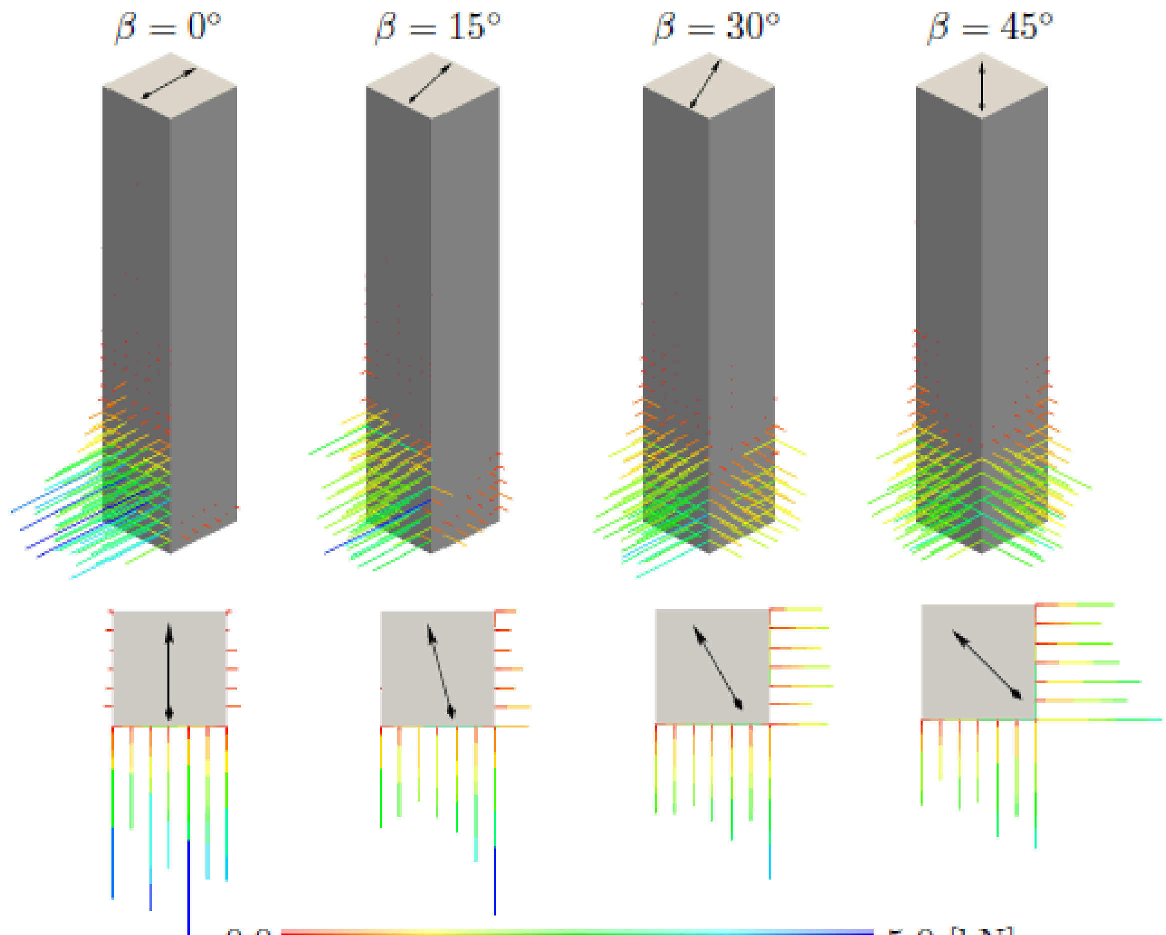
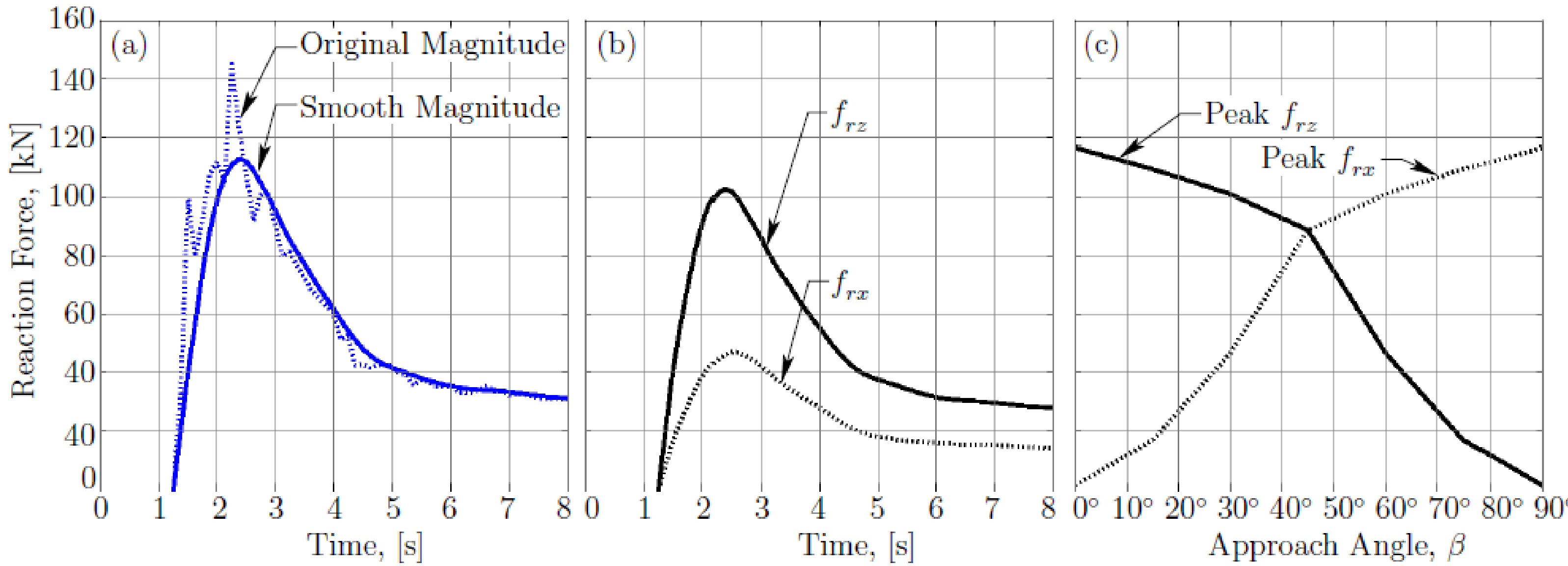
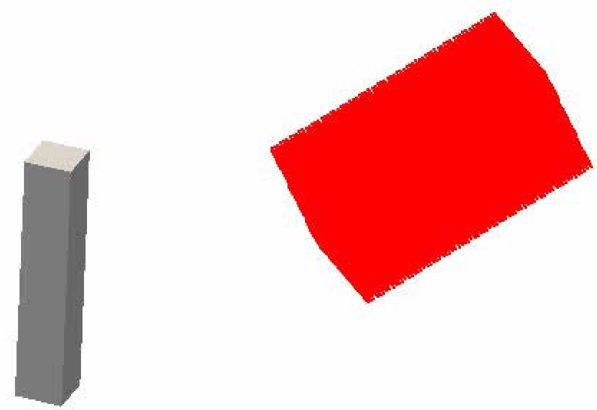


Applications – Effects of Approach Angle

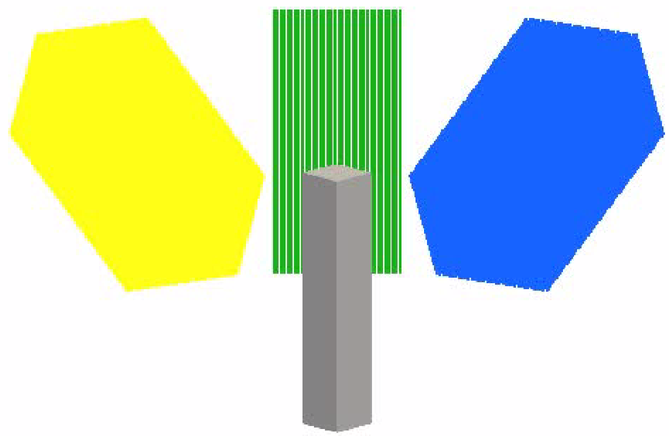


Applications – Effects of Approach Angle

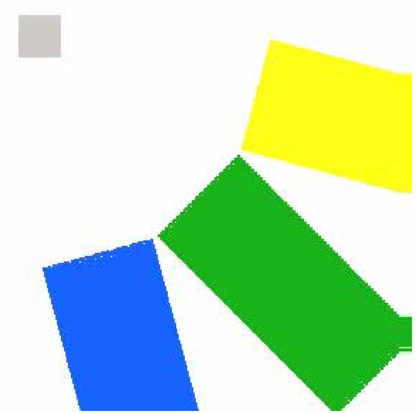


Questions ?

Applications – Effects of Approach Angle



Applications – Effects of Approach Angle



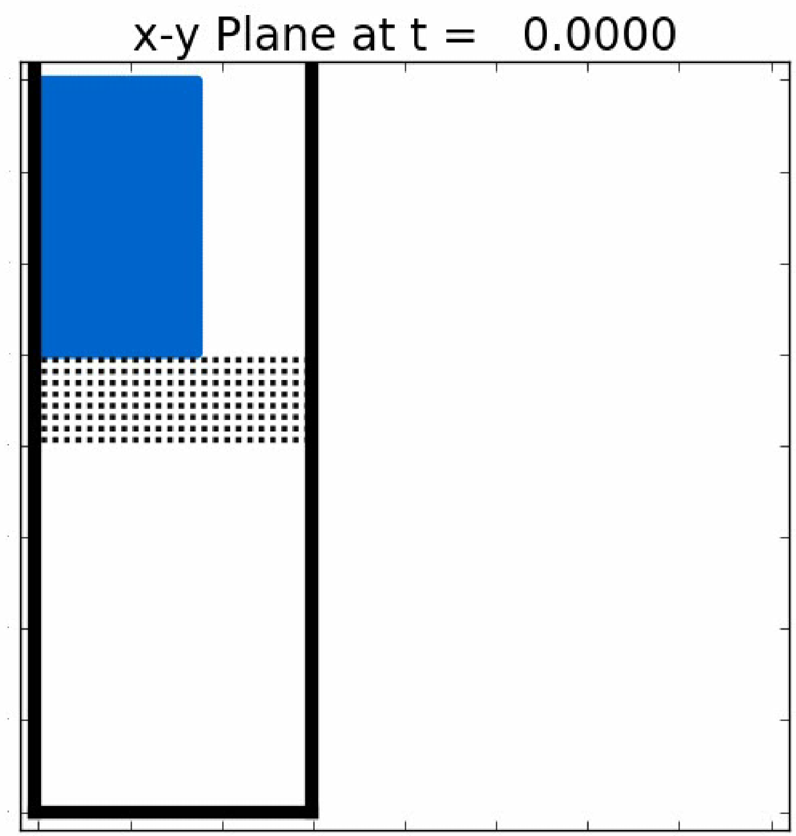
Conclusions

- Pedro's First Conclusion
  - Point 1
    - Subpoint 1

Questions ?

Research Goals

- Goals
  - Capture behavior associated with multiple phases



Material Point Method (MPM)

- Detailed formulation of the MPM
  - Weak form

$$\int_{V_{\mathcal{B}}} \delta\dot{\mathbf{v}} \cdot [\boldsymbol{\nabla} \cdot \boldsymbol{\sigma} + \mathbf{b} - \rho\,\dot{\mathbf{v}}]\; dV = 0$$

  - Approximation functions

$$\dot{\mathbf{v}} \approx \dot{\mathbf{v}}^h = \sum_i N_i\,\dot{\mathbf{v}}_i \qquad \delta\dot{\mathbf{v}}^h = \sum_i N_i\,\delta\dot{\mathbf{v}}_i$$

  - Particle-based integration

$$\int_{m_{\mathcal{B}}} (\bullet)\; dm \approx \sum_p (\bullet)_p\; m_p$$

Material Point Method (MPM)

- Detailed formulation of the MPM
  - Solving for nodal values

$$\sum_J m_{IJ}\,\dot{\boldsymbol{v}}_J = \boldsymbol{f}_I^{ext} + \boldsymbol{f}_I^{\sigma}$$

$$m_{IJ} = \sum_p N_I(\boldsymbol{x}_p)\,N_J(\boldsymbol{x}_p)\,m_p$$

$$\begin{aligned} \boldsymbol{f}_I^{ext} &= \sum_p \bar{\boldsymbol{b}}_p(\boldsymbol{x}_p)\,N_I(\boldsymbol{x}_p)\,m_p + \int_{\partial V_B} \tilde{\boldsymbol{t}}\,N_I(\boldsymbol{x})\,dS \\ \boldsymbol{f}_I^{\sigma} &= -\sum_p \boldsymbol{\sigma}_p \cdot \boldsymbol{\nabla} N_I(\boldsymbol{x}_p)\,m_p\ . \end{aligned}$$

Material Point Method (MPM)

- Detailed formulation of the MPM
  - Particle update (assume linear elastic material)

$$\bar{\boldsymbol{\sigma}}_{p,n+1} = \frac{\partial \bar{\psi}}{\partial \boldsymbol{\varepsilon}} = \bar{\mathbb{C}} : \boldsymbol{\varepsilon}_{p,n+1}$$

$$\boldsymbol{v}_{p,n+1} = \boldsymbol{v}_{p,n} + \sum_I N_I(\boldsymbol{x}_p)\, \Delta \boldsymbol{v}_I$$

$$\boldsymbol{x}_{p,n+1} = \boldsymbol{x}_{p,n} + \sum_I N_I(\boldsymbol{x}_p)\, \Delta \boldsymbol{x}_I$$

Material Point Method (MPM)

- Detailed anti-locking formulation
  - Volumetric Approach

$$\mathbf{M} := [\ 1\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0\ ]\ ,\ \mathbf{M}^{*} := [\ 1/3\ \ 1/3\ \ 1/3\ \ 0\ \ 0\ \ 0\ ]^{T} \quad \text{and} \quad \mathbf{S} := [1]$$

$$\boldsymbol{\alpha} := \{\,\hat{\theta}\,\}\,,\ \delta\boldsymbol{\beta} := \{\,\delta\hat{p}\,\}\,, \quad \text{and}\ \boldsymbol{\beta} := \{\,\hat{p}\,\}\,,\ \delta\boldsymbol{\alpha} := \{\,\delta\hat{\theta}\,\}\,.$$

Material Point Method (MPM)

- Detailed anti-locking formulation
  - Volumetric-Deviatoric Approach

$$\mathbf{M} := \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 2/3 & -1/3 & -1/3 & 0 & 0 & 0 \\ -1/3 & 2/3 & -1/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{M}^{*} := \begin{bmatrix} 1/3 & 1 & 0 & 0 & 0 & 0 \\ 1/3 & 0 & 1 & 0 & 0 & 0 \\ 1/3 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{S} := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \xi & \eta & \zeta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi & \eta & \zeta & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi & \eta & \zeta \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\alpha = \{\alpha_1\ \alpha_2\ \alpha_3\ \ldots\ \alpha_{15}\}^T \quad \text{and} \quad \beta = \{\beta_1\ \beta_2\ \beta_3\ \ldots\ \beta_{15}\}^T$$

Material Point Method (MPM)

- Detailed anti-locking formulation
  - Cell-Based Anti-Locking

$$\boldsymbol{\alpha}_c = \mathbf{H}_c^{-1}\,\mathbf{R}_c^{\tilde{\varepsilon}} \quad \text{and} \quad \boldsymbol{\beta}_c = \mathbf{H}_c^{-1}\,\mathbf{R}_c^{\tilde{\sigma}}$$

$$\begin{aligned}
\mathbf{H}_c &= \sum_{p\in c} \mathbf{S}_p^T\,\mathbf{M}\,\mathbf{M}^*\,\mathbf{S}_p\,m_p\ , \\
\mathbf{R}_c^{\tilde{\varepsilon}} &= \sum_{p\in c} \mathbf{S}_p^T\,\mathbf{M}\,\mathbf{M}^*\,\mathbf{M}\,\tilde{\boldsymbol{\varepsilon}}_p\,m_p\ , \qquad \text{and} \\
\mathbf{R}_c^{\tilde{\sigma}} &= \sum_{p\in c} \mathbf{S}_p^T\,\mathbf{M}\,\mathbf{M}^*\,\mathbf{M}\,\tilde{\boldsymbol{\sigma}}_p\,m_p
\end{aligned}$$

$$\boldsymbol{\varepsilon}_p^h = \mathbf{M}^*\,\mathbf{S}_p\,\boldsymbol{\alpha}_c + \tilde{\boldsymbol{\varepsilon}}_p - \mathbf{M}^*\,\mathbf{M}\,\tilde{\boldsymbol{\varepsilon}}_p$$

$$\bar{\boldsymbol{\sigma}}_p^h = \mathbf{M}^*\,\mathbf{S}_p\,\boldsymbol{\beta}_c + \tilde{\boldsymbol{\sigma}}_p - \mathbf{M}^*\,\mathbf{M}\,\tilde{\boldsymbol{\sigma}}_p$$

Material Point Method (MPM)

- Detailed anti-locking formulation
  - Node-Based Anti-Locking

$$\boldsymbol{\alpha}_i = \mathrm{H}_i^{-1}\,\mathrm{R}_i^{\tilde{\varepsilon}} \quad \text{and} \quad \boldsymbol{\beta}_i = \mathrm{H}_i^{-1}\,\mathrm{R}_i^{\tilde{\sigma}}$$

$$\begin{aligned}
\mathrm{H}_i &= \sum_{p\in i} \mathrm{S}_p^T\,\mathrm{M}\,\mathrm{M}^*\,\mathrm{S}_p\,N_{i,p}\,m_p\ , \\
\mathrm{R}_i^{\tilde{\varepsilon}} &= \sum_{p\in i} \mathrm{S}_p^T\,\mathrm{M}\,\mathrm{M}^*\,\mathrm{M}\,\tilde{\varepsilon}_p\,N_{i,p}\,m_p\ , \qquad \text{and} \\
\mathrm{R}_i^{\tilde{\sigma}} &= \sum_{p\in i} \mathrm{S}_p^T\,\mathrm{M}\,\mathrm{M}^*\,\mathrm{M}\,\tilde{\boldsymbol{\sigma}}_p\,N_{i,p}\,m_p
\end{aligned}$$

Material Point Method (MPM)

- Detailed anti-locking formulation
  - Node-Based Anti-Locking

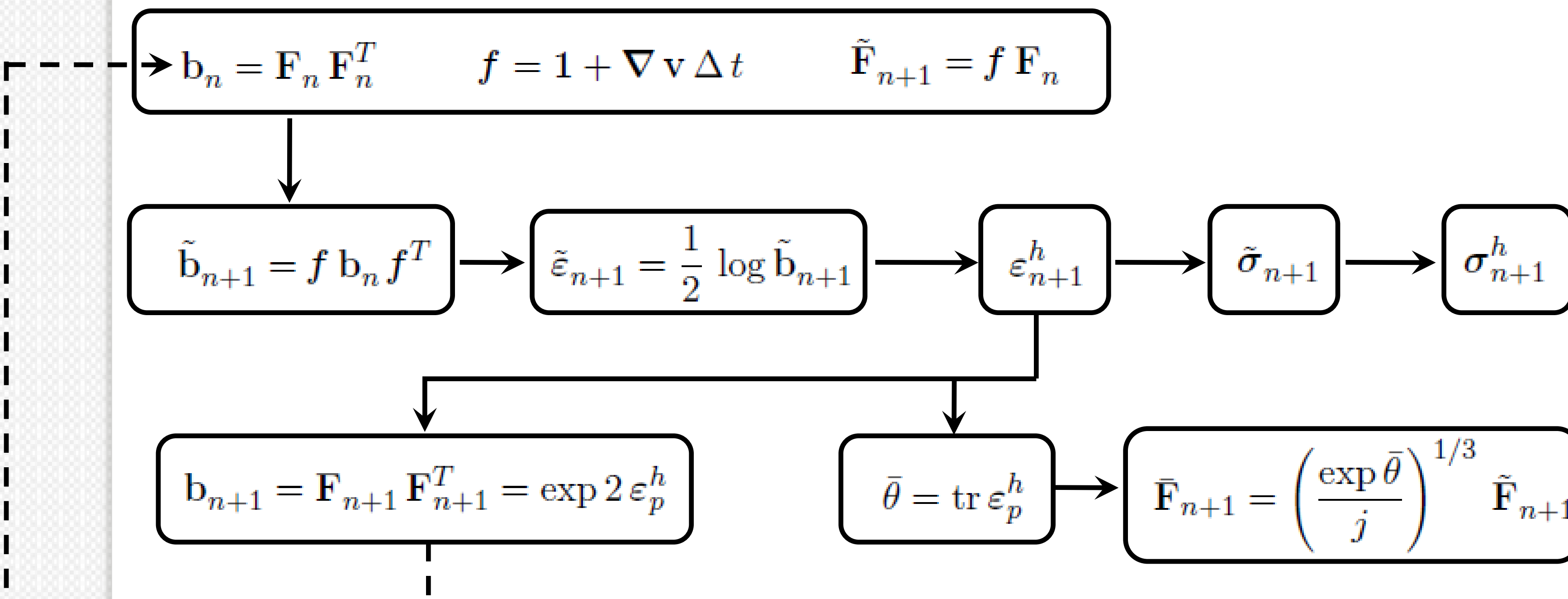
$$\tilde{\varepsilon}_i = \frac{\sum_{p\in i} \tilde{\varepsilon}_p N_{i,p} m_p}{\sum_{p\in i} N_{i,p} m_p} \quad \text{and} \quad \tilde{\boldsymbol{\sigma}}_i = \frac{\sum_{p\in i} \tilde{\boldsymbol{\sigma}}_p N_{i,p} m_p}{\sum_{p\in i} N_{i,p} m_p}$$

$$\begin{aligned} \varepsilon_i^h &= \mathrm{M}^* \mathrm{S}_i \alpha_i + \tilde{\varepsilon}_i - \mathrm{M}^* \mathrm{M} \tilde{\varepsilon}_i \\ \bar{\sigma}_i^h &= \mathrm{M}^* \mathrm{S}_i \beta_i + \tilde{\sigma}_i - \mathrm{M}^* \mathrm{M} \tilde{\sigma}_i \end{aligned}$$

$$\begin{aligned} \varepsilon_p^h &= \sum_i N_{i,p} \varepsilon_i^h \\ \bar{\sigma}_p^h &= \sum_i N_{i,p} \bar{\sigma}_i^h . \end{aligned}$$

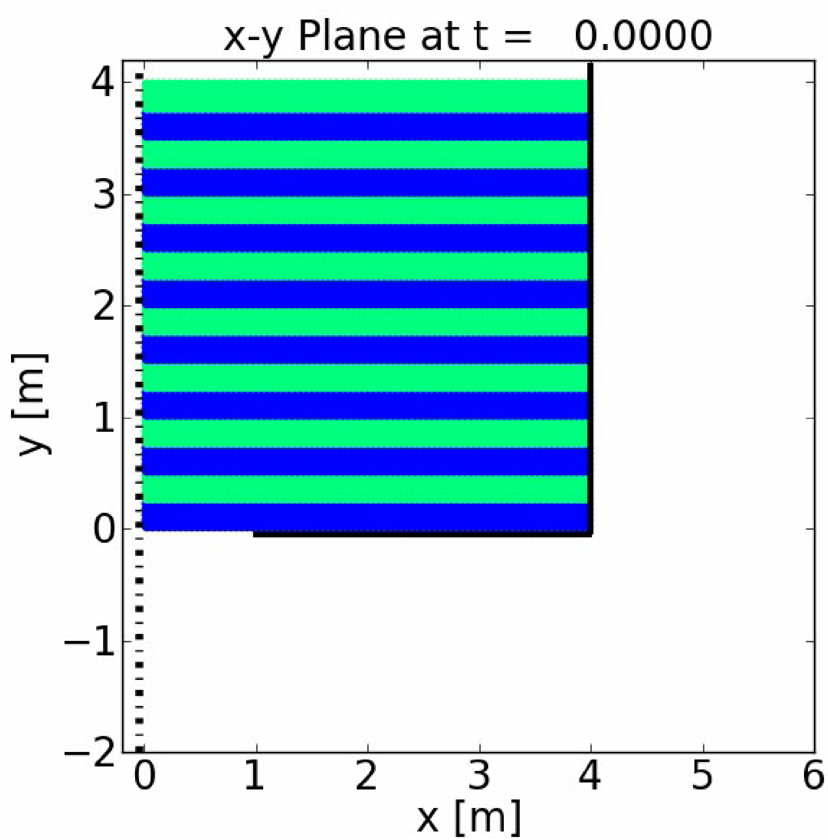
Material Point Method (MPM)

- Detailed anti-locking formulation
  - Large deformation flow chart



Anti-Locking Strategies in the MPM

- Mitigating Locking
  - Draining water tank (Standard MPM)



Anti-Locking Strategies in the MPM

- Mitigating Locking
  - Draining water tank (Cell-based anti-locking)