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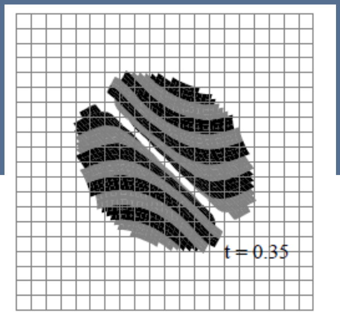
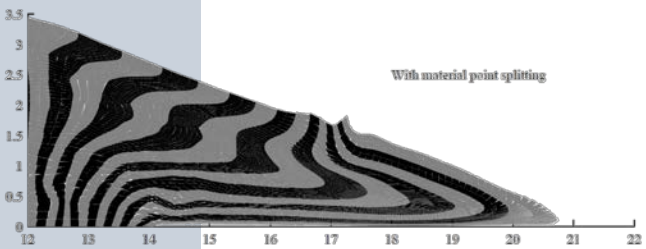
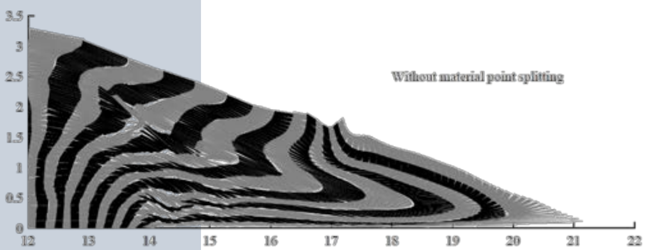
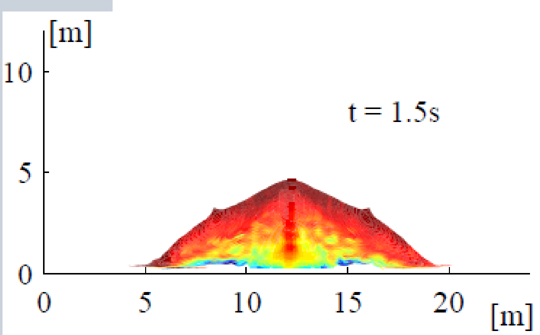








Post-processing for the Material-Point Method

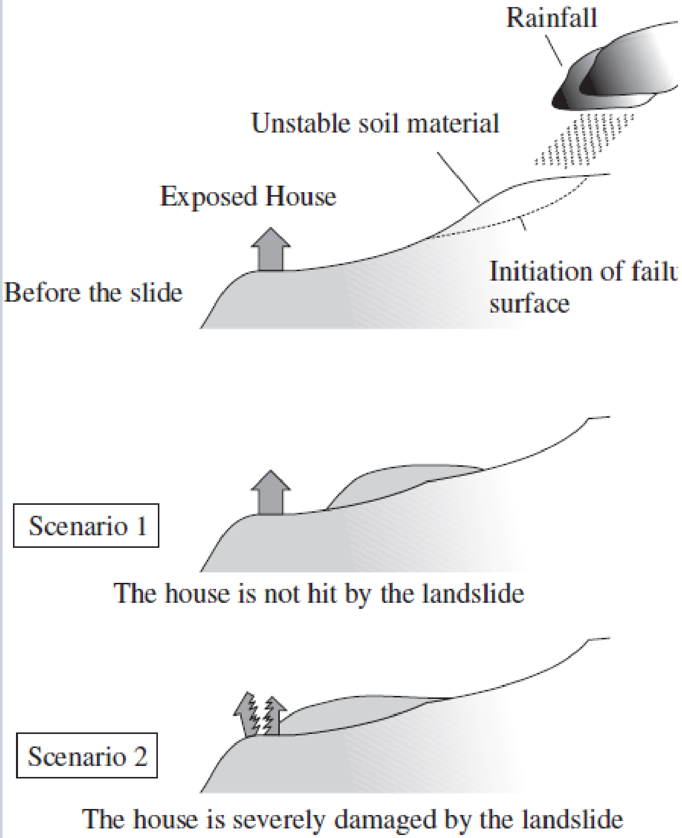
- *And adaptive discretization by splitting of material points*

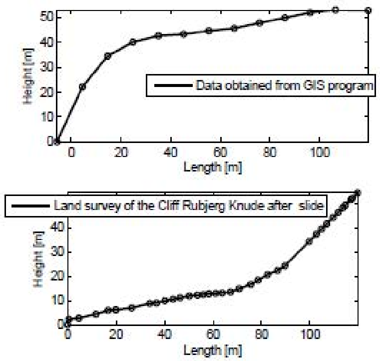
Søren Mikkel Andersen

Marts 14-15th, 2013 at 7th MPM workshop, Salt Lake City, Utah

Department of Civil Engineering,
Aalborg University, Denmark

Motivation for starting MPM research





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Program:

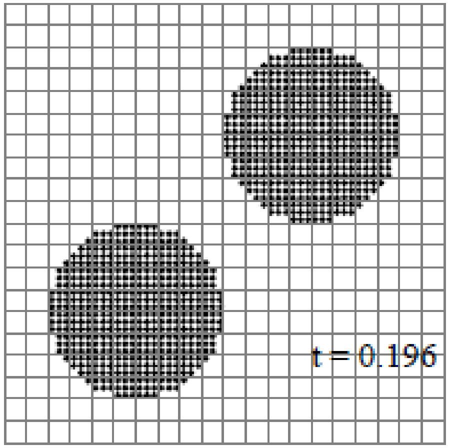
Visualization of deformation for the material-point method – and other point based methods

Postprocessing applied to the stress field within MPM

Adaptive discretication within MPM

Conclusions

New technique for visualization of deformation for the material-point method – and other point based methods



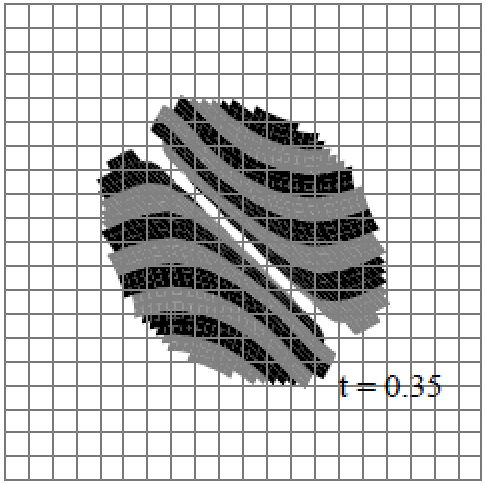
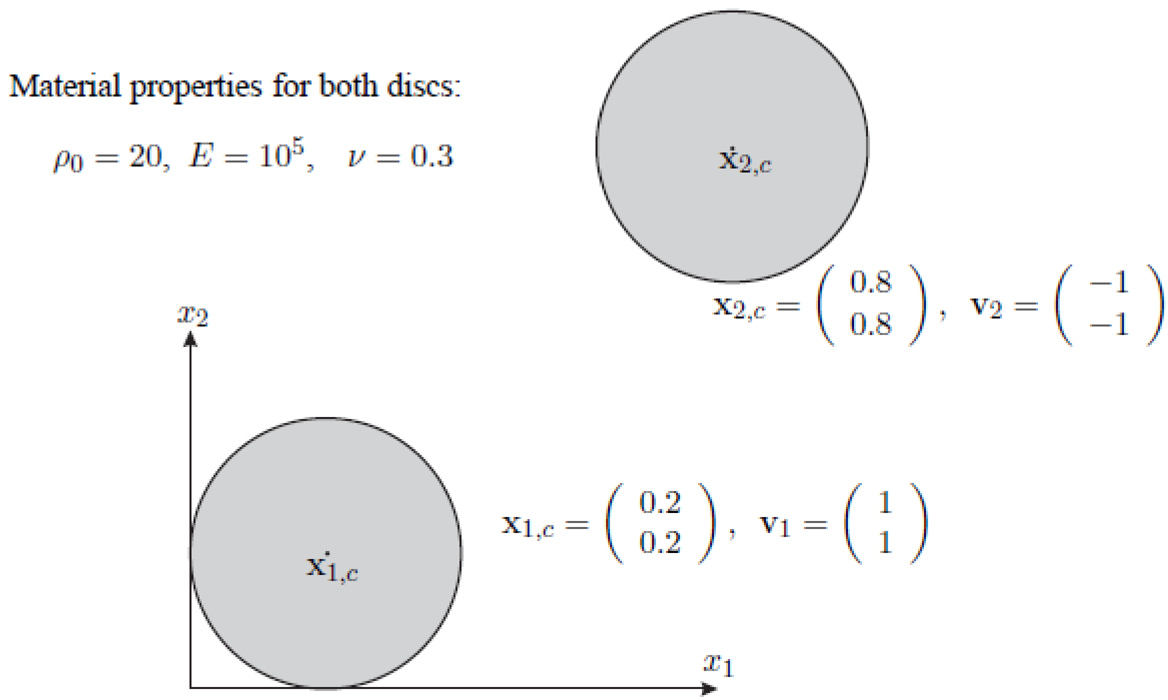


Illustration of MPM-results in – *Colliding disc problem revisited*



Traditional MPM visualization – Material points are shown as points/small dots

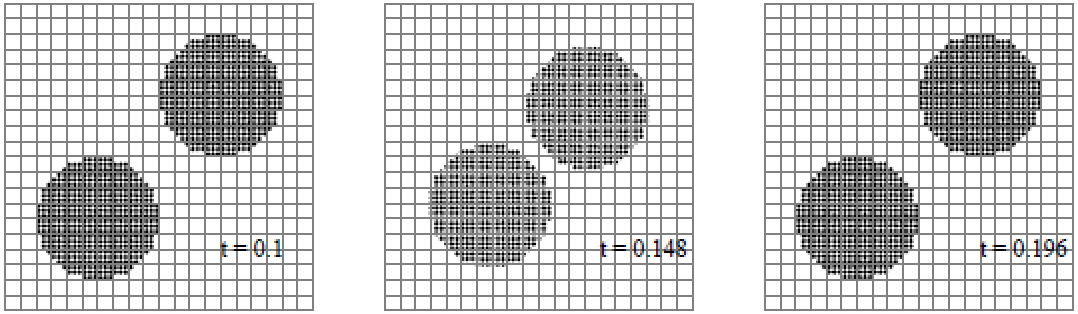


Fig. 4. Position of material points as a function of time. The discs collide and then bounce off.

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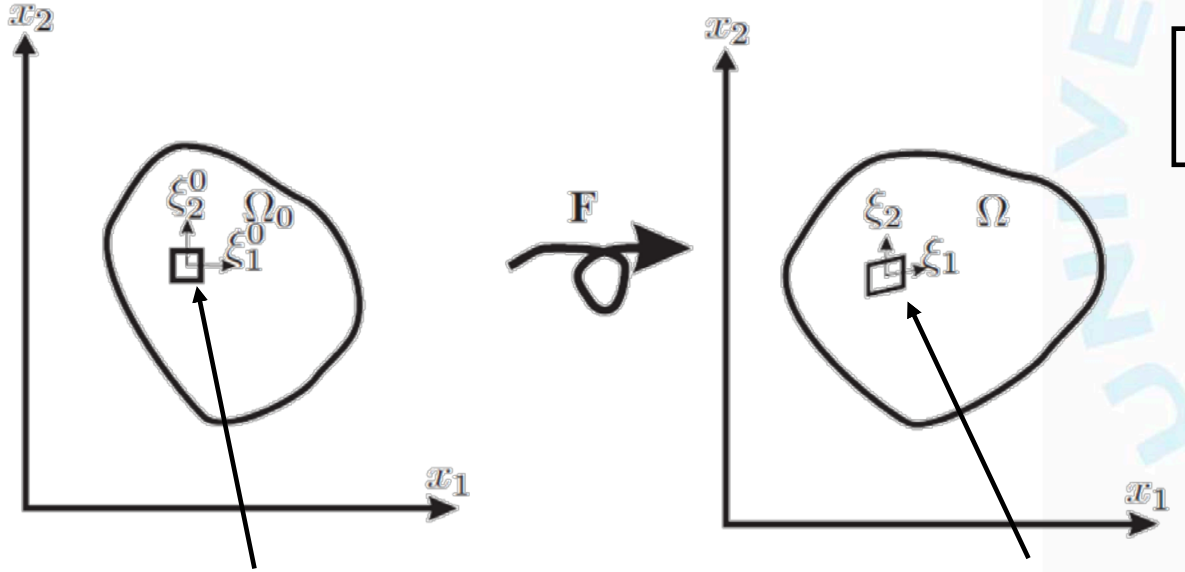
Visualization of large deformations

A material point represents part of a domain which undergoes both movement and deformation.
A new way of visualizing large deformations is obtained by introducing the deformation gradient tensor as state variable for the material points:

$$\mathbf{F}_p(t) = \frac{\partial \mathbf{x}_p}{\partial \mathbf{x}_p^0}$$

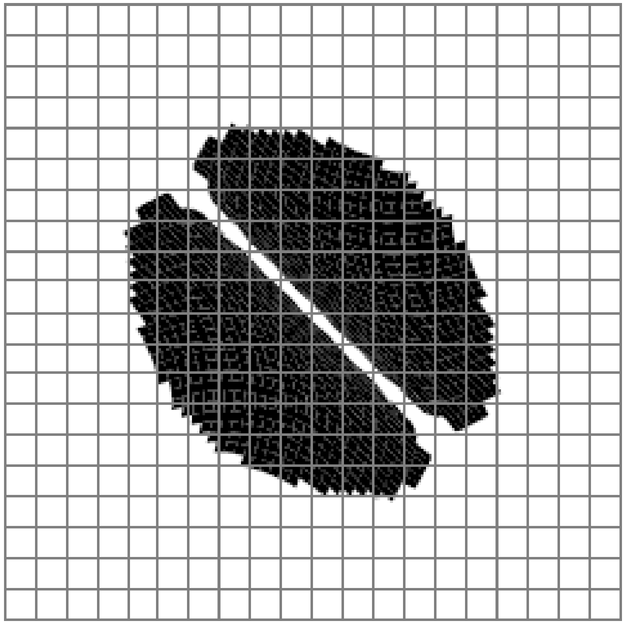


The material point is initially assigned a rectangular part of the physical domain, a *voxel*

The *deformed configuration* of the voxel is determined using the deformation gradient

Colliding discs – Tracking large deformations using the deformation gradient tensor and

The two discs are now modelled as Tresca Materials with a cohesion of *c=5*

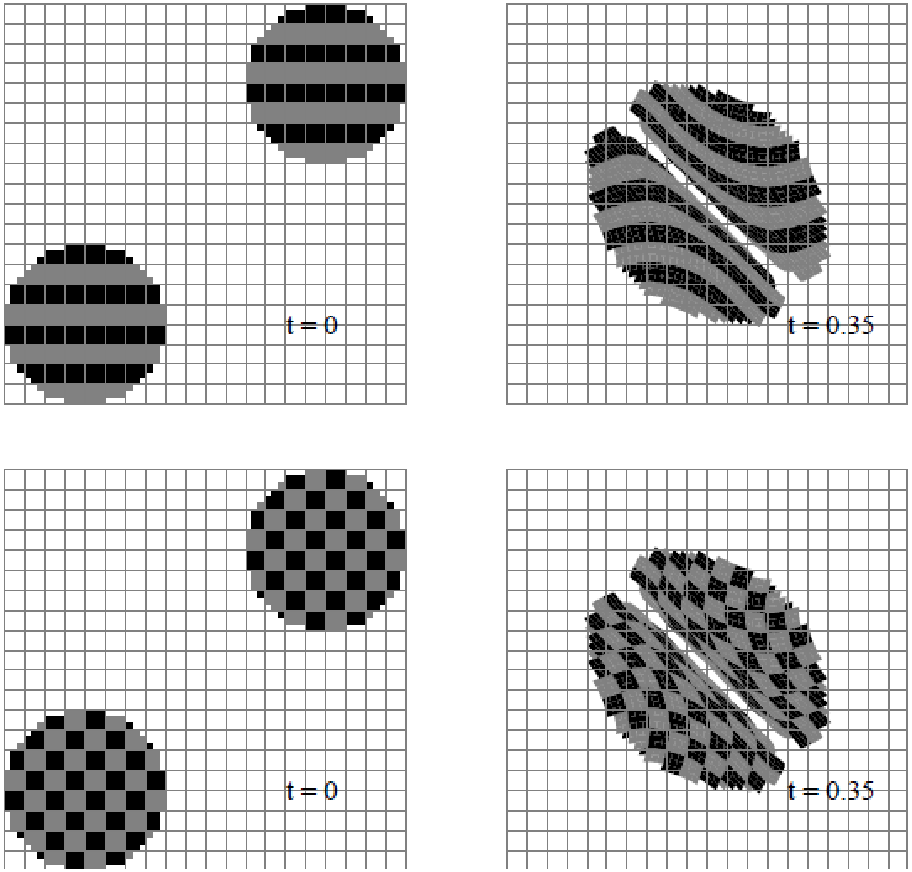


1. The deformation gradient is tracked through the MPM-cycle

2. The deformed (initially rectangular) voxels are calculated using the deformation gradient

Disclaimer: These capabilities to visualize the deformation can be added to any type of two-dimensional MPM code

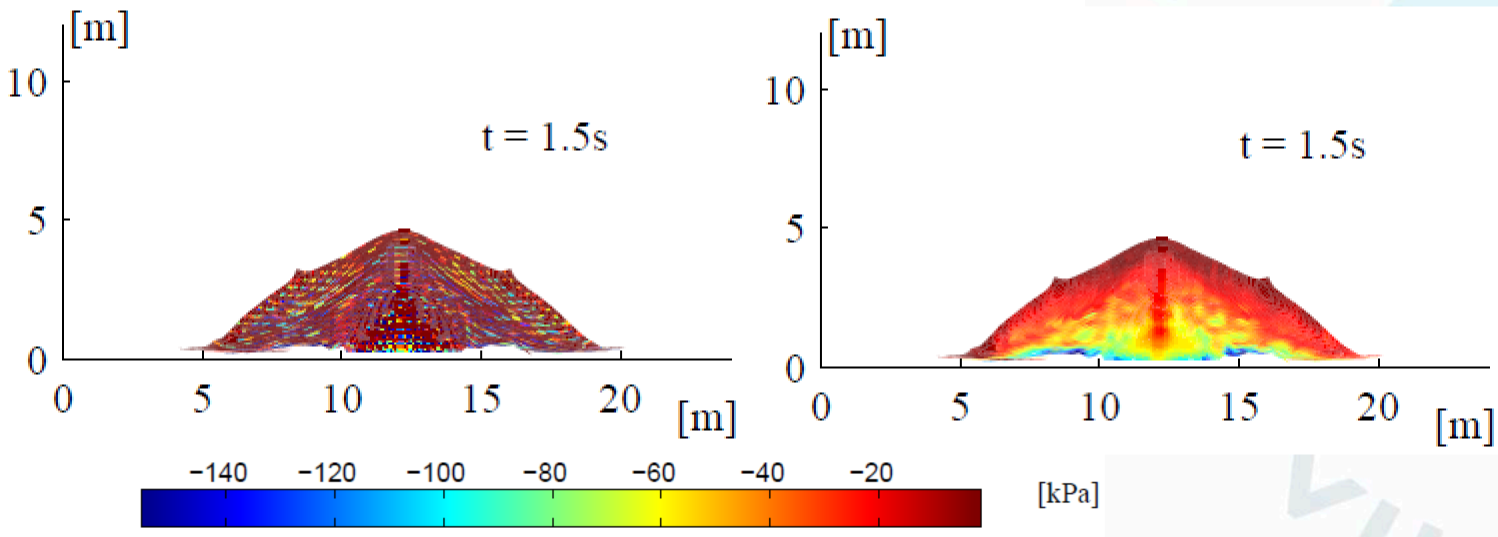
Simple technique for improving the visualization of large deformation – coloration schemes



The visualization of large deformation can be further improved by assigning a pattern of dummy colours to the voxels

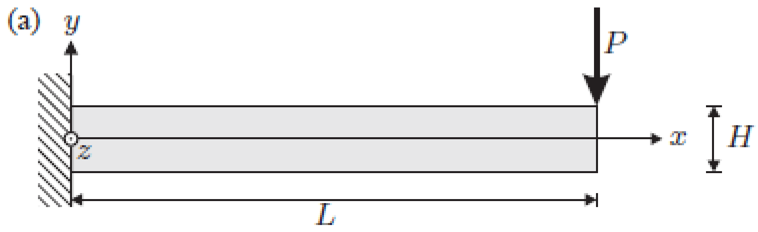
This type of enchancing the description of deformation could be very easy to implement for other point based method such as Smoothed particle hydrodynamics (SPH) by adding a deformation gradient tensor and a voxel for each particle

Post-processing applied to the stress field within MPM



Horizontal normal stresses during a collapse of a column of soil

Problematic stresses in MPM - *Cantilevered beam subjected to a point force*



length $L = 8$ and the height $D = 1$
$E = 3 \cdot 10^5$, $\nu = 0$ and $\rho_0 = 10^3$.

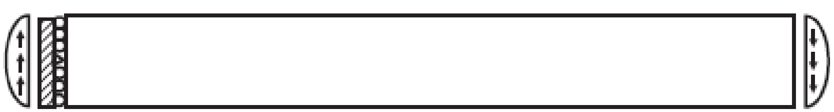
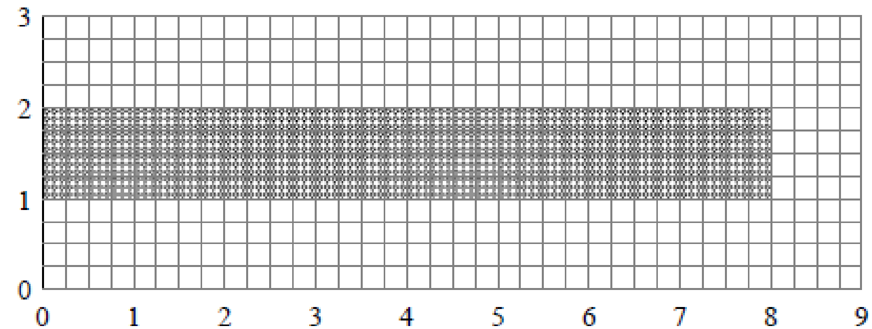
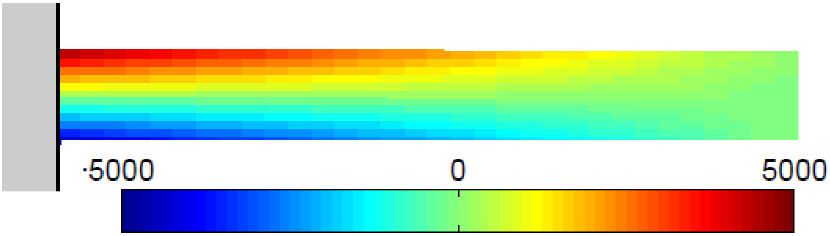


Figure 6. The boundary conditions applied in the numerical study.

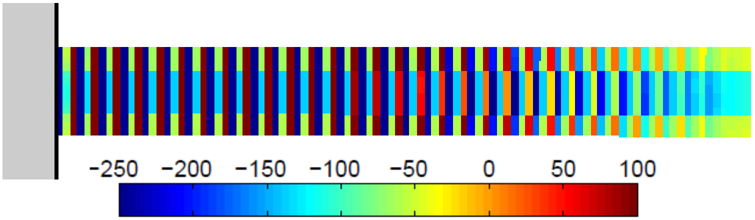


The cantilevered beam problem – stress fields at the material points



Horiontal normal stresses – generally they are well behaved and correct – According to the Naviers formular

Simulation performed with original MPM formulation with a linear finite element grid



Shear stresses – Ill behaved !

The cantilevered beam problem – stress fields at the material points

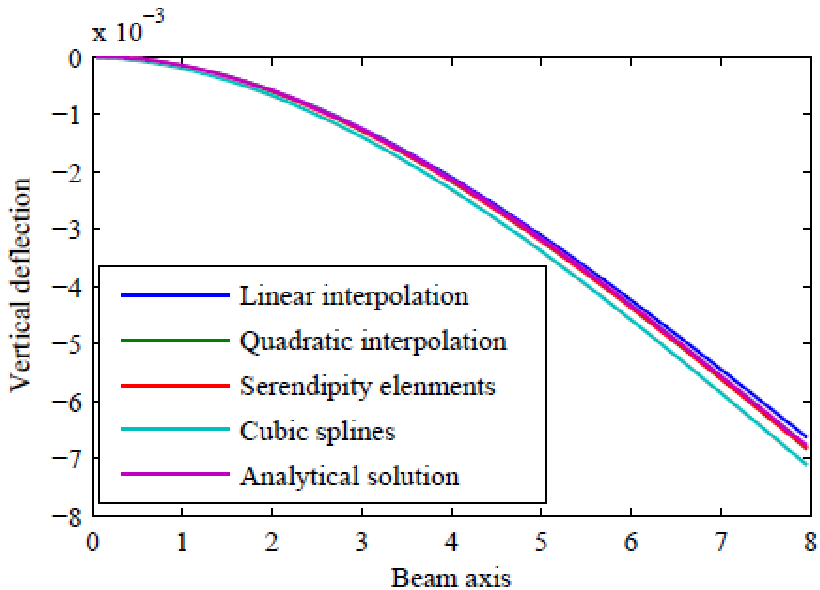
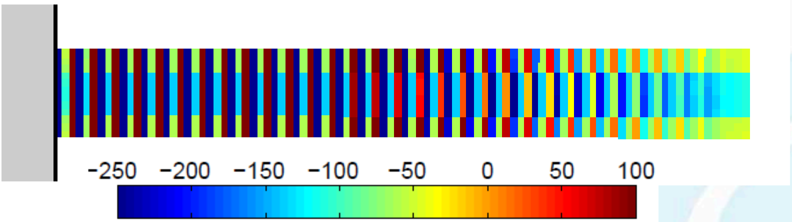


Figure 13. Vertical deflection of the beam neutral axis at the end of the different MPM simulations compared with the analytical solution.



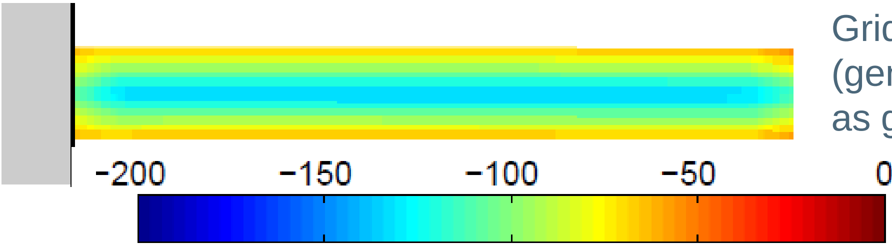
Shear stresses – Its a complete mess !

The vertical deflection matches the analytical solution despite severerly distorted stresses – Somewhat of a paradox ?

The cantilevered beam problem – stress field grid nodes as a post-processing

$$\boldsymbol{\sigma}_i = \sum_{p=1}^{N_p} \frac{\boldsymbol{\sigma}_p \Phi_{ip} m_p}{m_i}$$

The stress tensor for grid node *i*

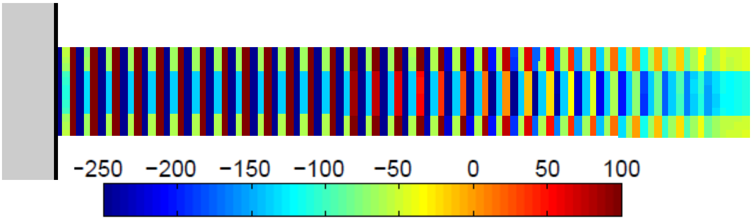


Grid node shear stress field using. The (generally) correct shear stress distribution as given by Grashoffs formula is obtained.

IE: Despite that the stresses, $\boldsymbol{\sigma}_p$ are very erradic, the stresses when interpolated to the grid nodes, $\boldsymbol{\sigma}_i$ are physically realistic.This is a contradiction the claim that the material-point method is a mesh-free method – in this case the mesh is where the physics is well kept.

σ_p is found to be very erradic – σ_i is found to be physically realistic – what is the problem ?

The constitutive equations are solved at the material points. Within study of geotechnical prolems and landslides etc., shear failure , strain softening, localization of deformation along shear band are all important physical features.



Shear stresses – Its a complete mess !

This is an isssue of concern for elasto-plastic materials, especiallly if the plastic behaviour is due to shear and the plastic deformation is localized.

Reproduced with permission from the work of Eilertsen et al. (2008)

Eilertsen, RS, Hansen, L, Bargel, TH, and Solberg, IL (2008). Clay slides in the Målselv valley northern Norway: Characteristiscs, occurence, and triggering mechanicms. *Geomorphology* **93**, 548–562.

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Stress fields for finite strain problems

For finite strain problems, and problems where the geometry is complicated, the geometry is associated with the set of material-points and its not possible to visualize meaningfull grid stresses. Instead a post-processing extraction of a smoothed MPM-stress field is suggested:

1. Grid-node stresses are defined by:

$$\boldsymbol{\sigma}_i = \sum_{p=1}^{N_p} \frac{\boldsymbol{\sigma}_p \Phi_{ip} m_p}{m_i};$$

2. Smoothed material-point stresses are extracted by

$$\boldsymbol{\sigma}_p^{smooth} = \sum_{i=1}^{N_n} \boldsymbol{\sigma}_i \Phi_{ip}$$

Numerical Example – Collapse of an elasto-plastic soil-column

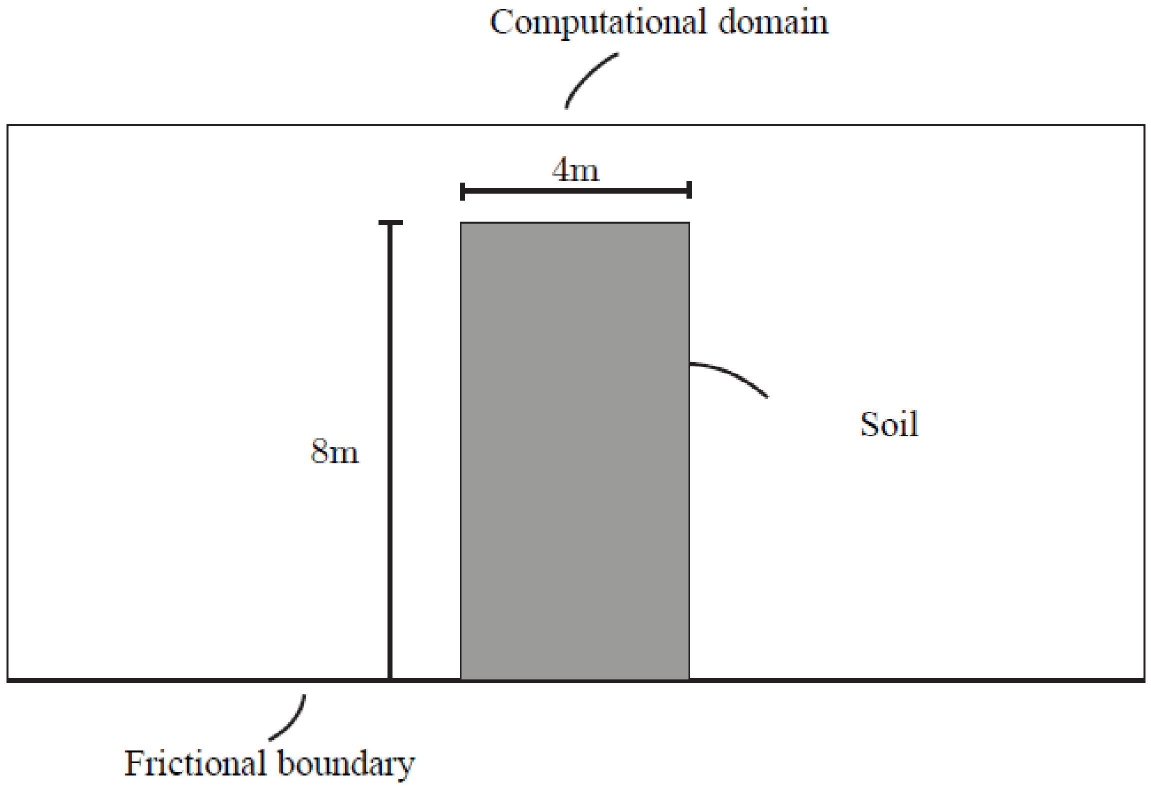
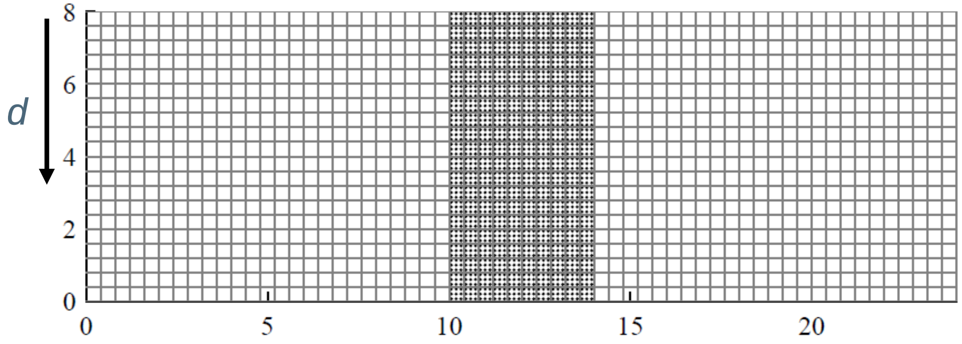


Illustration of material-point splitting
Collapse of a soil column

Soil: Mohr-Coulomb material

$$E = 20\text{MPa}, \quad \nu = 0.42, \quad \rho_0 = 10^3\text{kg/m}^3, \quad c = 1\text{kPa}, \quad \phi = 42^\circ \quad \text{and} \quad \psi = 0^\circ$$



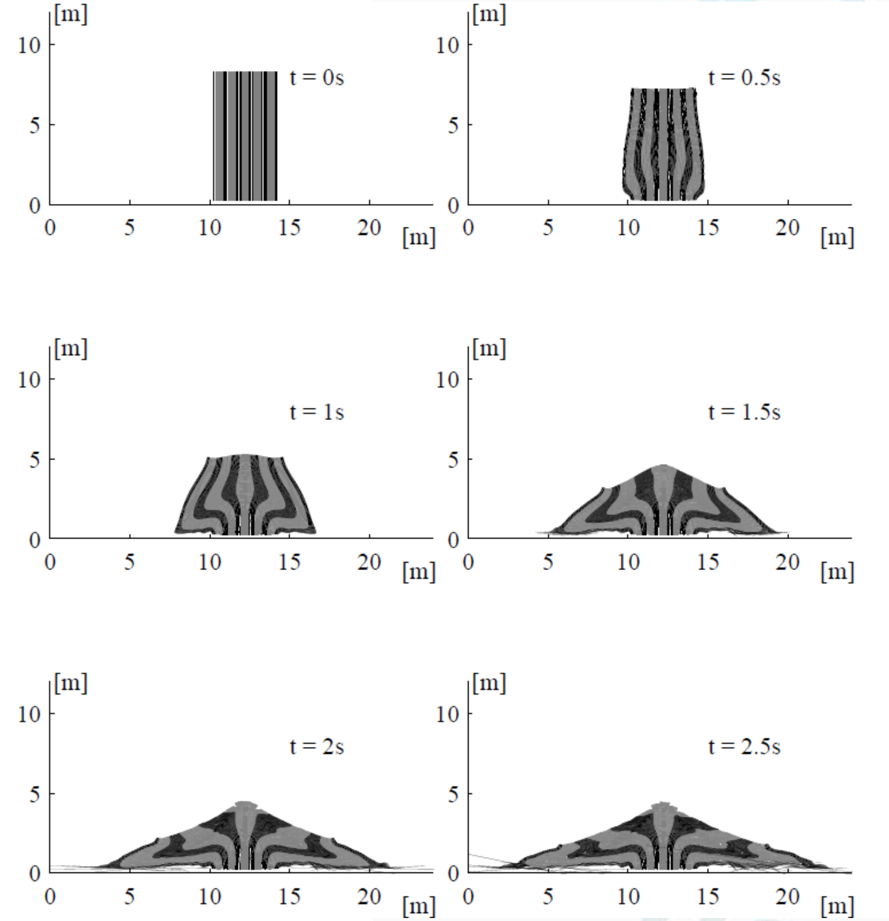
Initial stress field:

$$\sigma_{yy}^0 = -dg\rho^0,$$

$$\sigma_{xx}^0 = \sigma_{zz}^0 = -dg\rho^0 K_0,$$

$$K_0 = \nu/(1-\nu).$$

Collapse of an elasto-plastic soil-column
- Deformation pattern

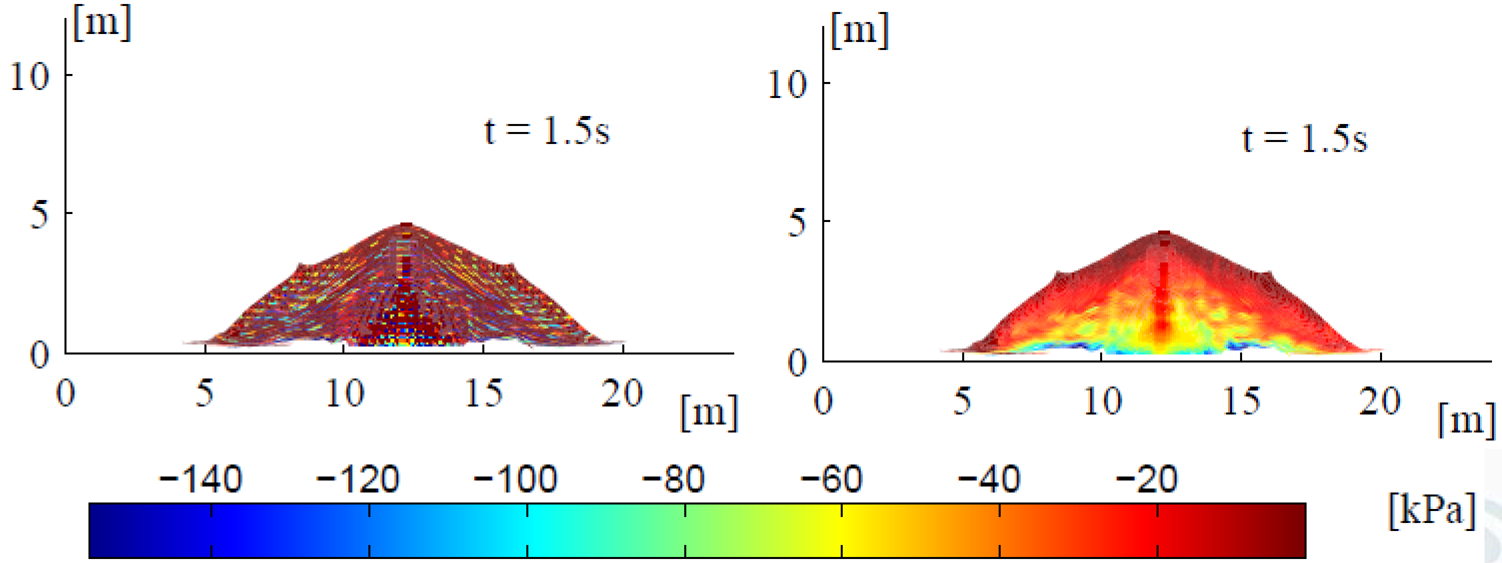


Collapsing soil column –
Extraction of a smoothed stress field

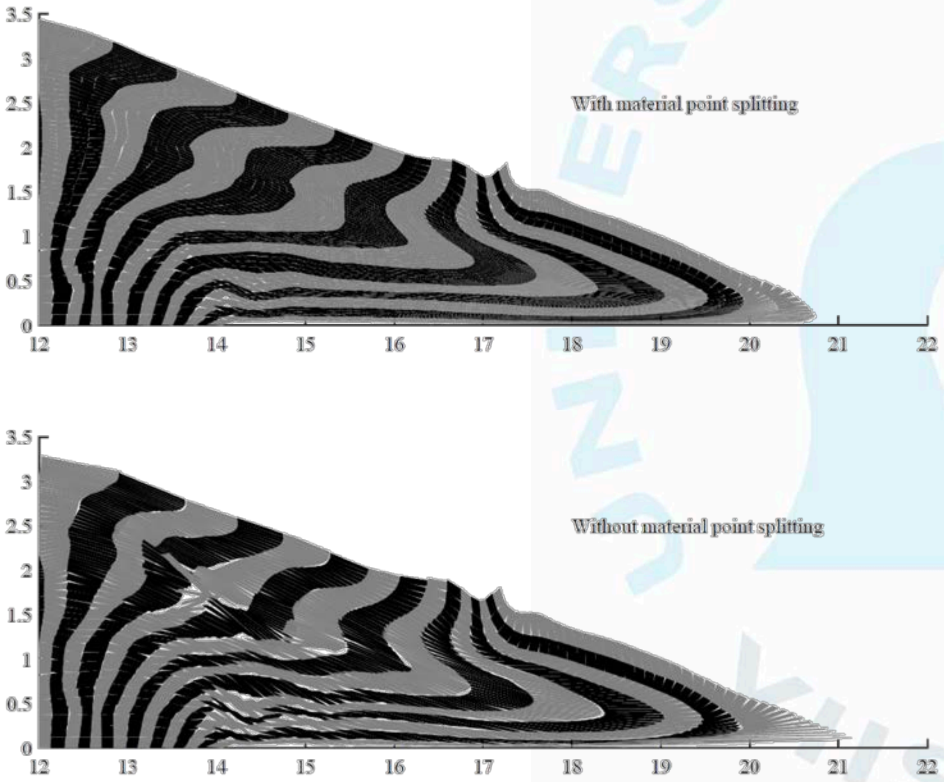
Vertical normal stress obtained as the postprocessing operation:

$$\boldsymbol{\sigma}_i = \sum_{p=1}^{N_p} \frac{\boldsymbol{\sigma}_p \Phi_{ip} m_p}{m_i} \qquad \boldsymbol{\sigma}_p^{smooth} = \sum_{i=1}^{N_n} \boldsymbol{\sigma}_i \Phi_{ip}$$

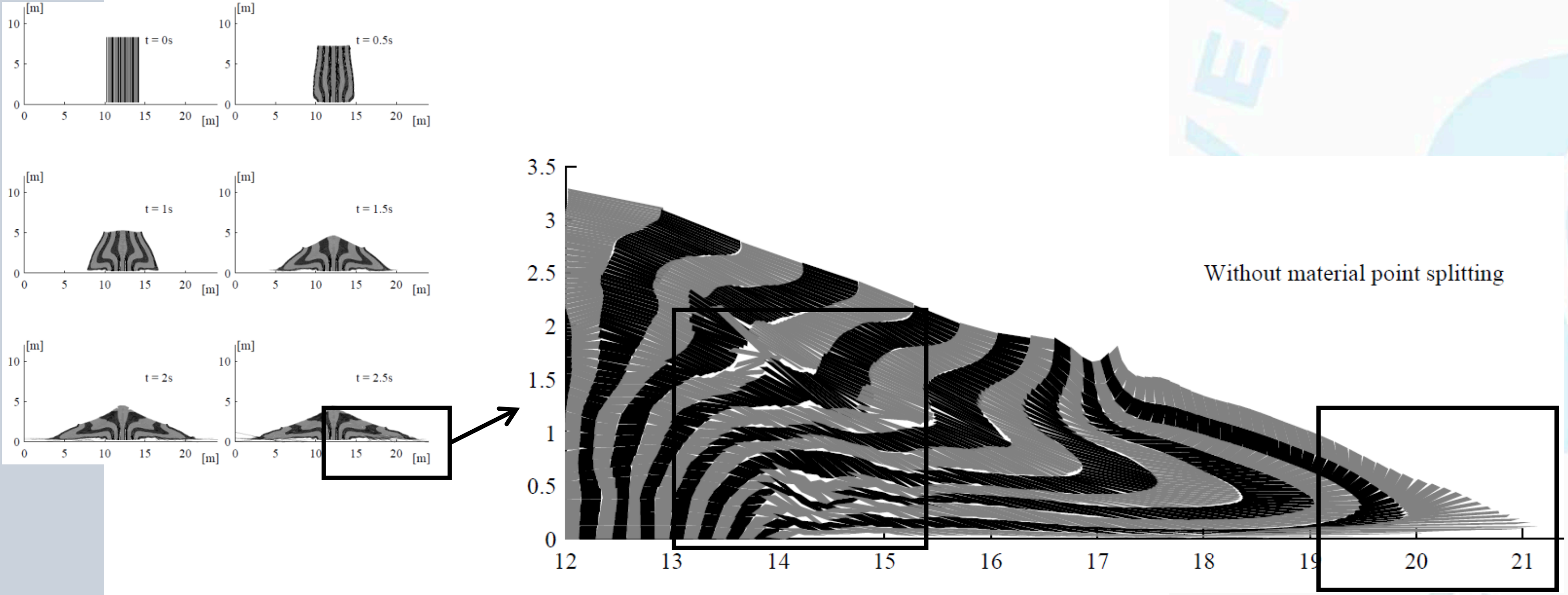
Vertical normal stresses obtained directly at the material points



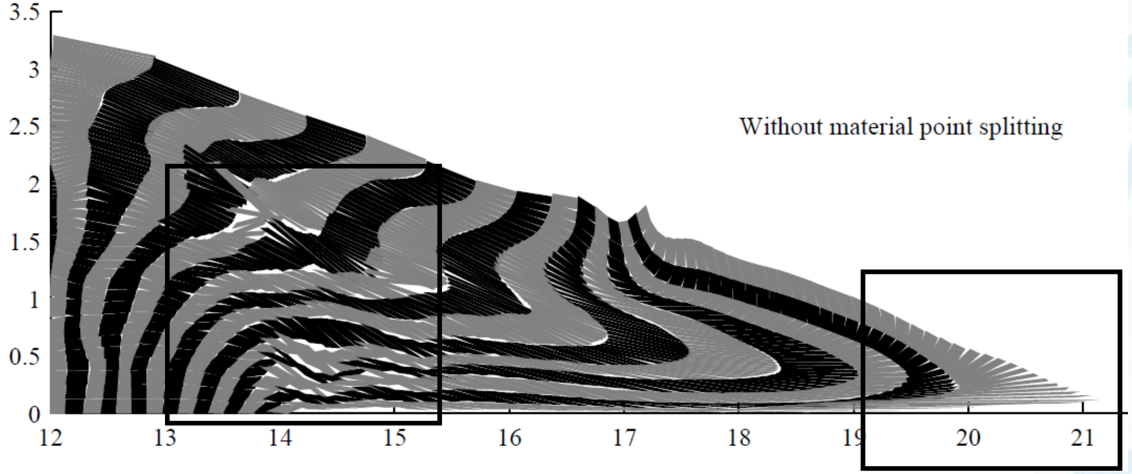
Adaptive discretization for the material-point method



Problem discovered from the tracking of voxels – unrealistic MPM-deformation – some material points become severely distorted



Concept – Adaptive discretization within the Material-Point method



Idea: When a material point becomes severely distorted, it can be split into two or more new material points

Adaptive discretization by material-point splitting

A variant of MPM is the generalized material point method (GIMP), where the interpolation functions and its gradients are given by

$$\bar{N}_{ip} = \frac{1}{V_p} \int_{\Omega_p \bigcap \Omega_i} N_i \chi_p dV$$

$$\frac{\partial \bar{N}_{ip}}{\partial \mathbf{x}} = \frac{1}{V_p} \int_{\Omega_p \bigcap \Omega_i} \frac{\partial N_i}{\partial \mathbf{x}} \chi_p dV.$$

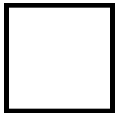
Further, material properties are often presumed to be constant within the domain of the material point as

$$\chi_p(\mathbf{x}_p) = \begin{cases} 1 & \text{if} \quad \mathbf{x} \in \Omega_p \\ 0 & \text{otherwise.} \end{cases}$$

Utilizing this, the volume associated with a material point may equally well be represented by two volumes, where each volume is assigned a new material point. In geneal , the GIMP interpolation functions are approximated so, there may be an effect of this when material points are split.

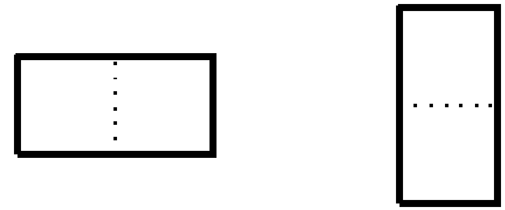
Illustration of splitting modes

Size of original material point

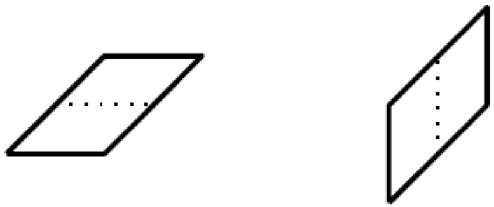


Conceptual illustration of a splitting algorithm using the deformation gradient tensor components as a criterion from an initially quadratically shaped voxel

$$F = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \qquad F = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$



$$F = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad F = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$



$$F = \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix} \qquad F = \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}$$

A simple splitting criterion

At the start of the MPM simulation:

$$\mathbf{F}_p^0 = \mathbf{F}_p^{split,0} = \mathbf{I}$$

A material point is split into two new ones whenever:

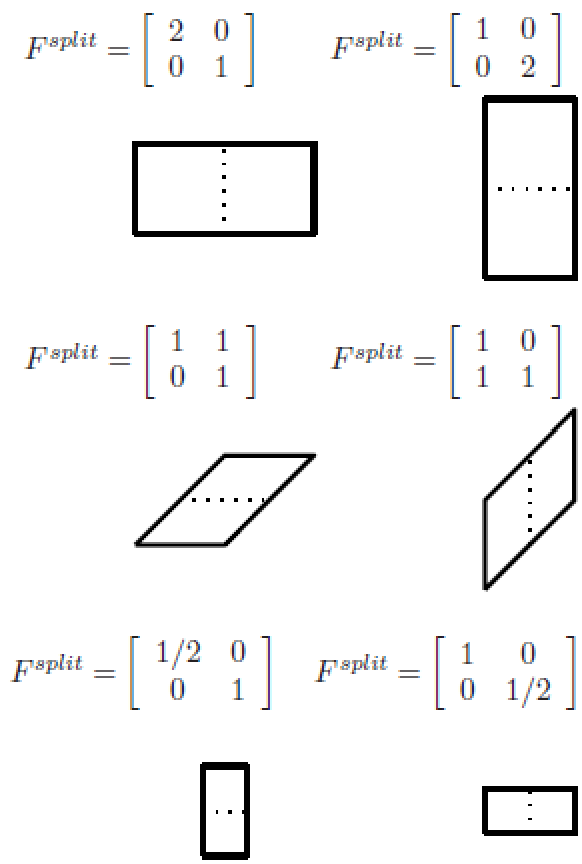
$$F_{11}^{split} > \alpha, \quad F_{22}^{split} > \alpha,$$

$$F_{11}^{split} < \beta, \quad F_{22}^{split} < \beta,$$

$$|F_{12}^{split}| > \gamma, \quad |F_{21}^{split}| > \gamma,$$

When a material point is split, state variables (velocity, stress, strain etc.) is the same as the original material point , while the mass is equally distributed into the two new ones.

The position of each material point is defind in the centroid of each of the 2 new voxels

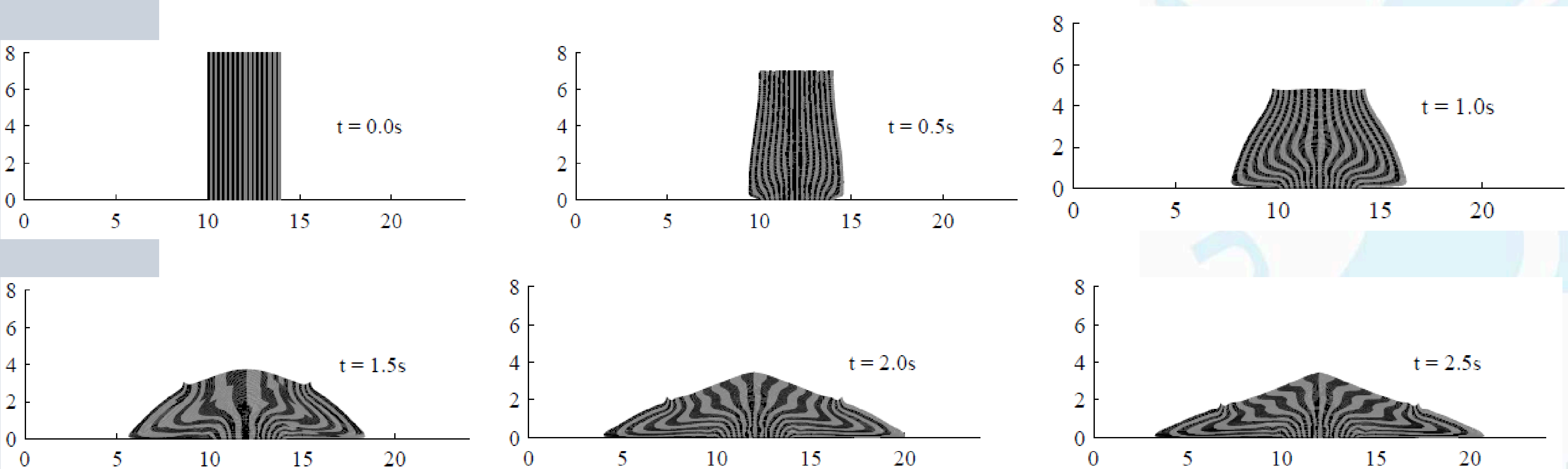


Collapsing soil column – With a splitting criterion applyed

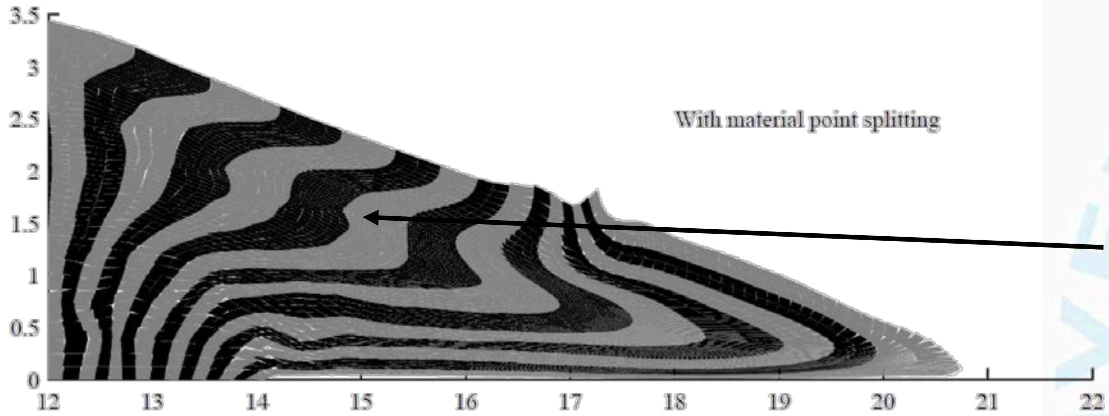
$$F_{11}^{split} > \alpha, \quad F_{22}^{split} > \alpha, \qquad \alpha = 2,\ \beta = 1/2 \text{ and } \gamma = 1$$

$$F_{11}^{split} < \beta, \quad F_{22}^{split} < \beta,$$

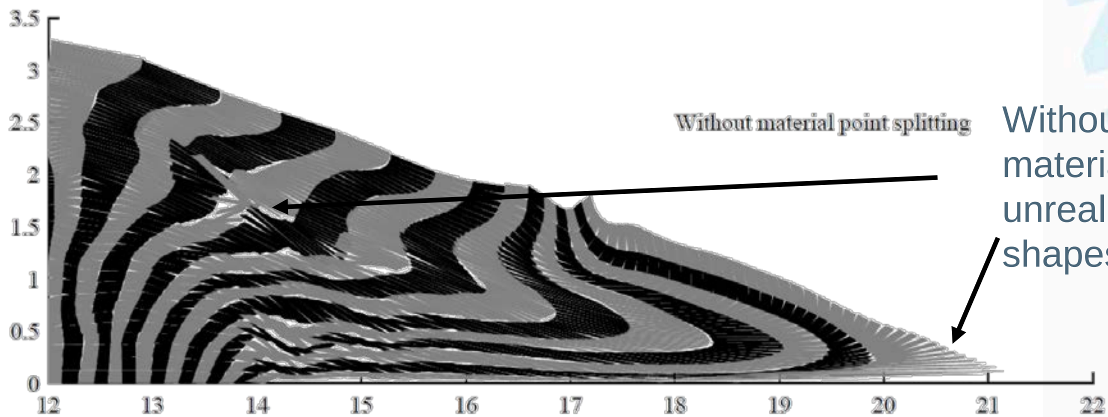
$$|F_{12}^{split}| > \gamma, \quad |F_{21}^{split}| > \gamma,$$



Comparison: With and without splitting of material points



The proposed splitting scheme improves the behaviour for problems involving extreme deformations



Without MP-splitting, material points obtain unrealistic ’string-like’ shapes

Untested idea - Performing-pre- runs to find the optimum discretization

For a particular combination of material (properties) and geometry, it may not be known a priori where the in the slope a potencial collapse will start. It would be benecial to have the finest discretization where the interesting physics are taking place. This could be done by.

1. An MPM pre-run is performed with a relatively coarse global discretization

2. Its identified where the critical parts of the model are – -

3. the original model is discarded , and a new model is generated employing the adaptive discretization - This process may be repeated as many times as desired ed finer, and both mesh and material points could be subjected to a rediscretization

4. A full MPM run in time is performed for the adaptively discretized model
   (Part 1-3 can be repeated as many times as desired.)

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Conclusions

1. The deformation gradient tensor should be tracked as part of the MPM-process if not done already - amongst to provide a tool to inspect the deformations visually

2. New properties discovered about the stresses within the material point method – Despite erratic stresses of the individual material points, physically realistic stresses can be extracted using the computational grid - this is also a reason for concern

3. A new concept is presented – to make automatic adaptive discretization by splitting material points into new ones in cased of localized discretization

4. I am very interested in doing coorporation in the future with your guys with the aim of improving the method

Thank you for your attention

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