Slides of Lecture 5
(Presented over two periods)

Today's Class: Energy Methods

- **Lagrange Equations**
- **Hamilton's Principle**

There are two energy methods we shall cover in this class. Together they can form the basis of deriving governing equations and discretized governing equations for almost any system we encounter

In this class and next, we shall derive the Lagrange equations and practice applying them. After that we shall examine Hamilton's principle.

Lagrange Equations: Theme

Up to now We have assembled the governing equations multi-body systems using:

- **force balance**
- **displacement continuity**
- **perspicacity**

We had to be smart.

Lagrange Equations Provide a Recipe

We do not have to be so smart.

We have broader choice of degrees of freedom - called generalized coordinates.

We can accommodate constraints, implicitly or explicitly.

Certain invariance properties hold (explained later).

Lagrange Equations: Derivation Outline for Systems of Particles

Newton's Law of Motion for a Particle:

$$\vec{F} = \frac{\partial}{\partial t}\vec{p}$$

where $\vec{p}$ is linear momentum.

Assuming constant mass and in the special case of a orthogonal Cartesian coordinate system,

$$F = \frac{\partial}{\partial t}\vec{p} = \frac{\partial}{\partial t}m\vec{v} = m\sum_{n=1}^{3}\ddot{x}_n\vec{b}_n =$$

where $\vec{b}_n$ are unit basis vector that are constant in time and space. This permits us to write a simple expression involving the components:

$$F_n = m\ddot{x}_n$$

Lagrange Equations: Derivation Outline for Systems of Particles

The big lesson here is that

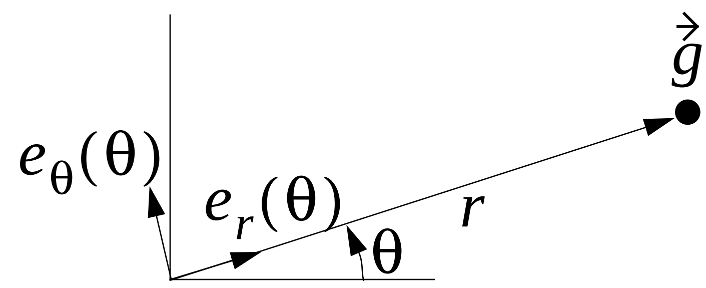
$$\vec{F} = \frac{\partial}{\partial t} m\dot{\vec{x}} \Rightarrow F_n = m\ddot{x}_n$$

for orthogonal, Cartesian coordinate systems

What about other coordinate systems?

Lagrange Equations: Derivation Outline for Systems of Particles

Consider a particle whose position, $\vec{g}$, is expressed in terms of a polar coordinate system: $\vec{g} = r\vec{e}_r(\theta)$



where the basis vectors $\vec{e}_r(\theta)$ and $\vec{e}_\theta(\theta)$ are related to the standard orthogonal cartesian coordinate system by: $\vec{e}_r(\theta) = \cos(\theta)\vec{i} + \sin(\theta)\vec{j}$ and

$$\vec{e}_\theta(\theta) = -\sin(\theta)\vec{i} + \cos(\theta)\vec{j}$$

The particle moves, so the coordinates r and θ of the particle are functions of time: $r = r(t)$ and $\theta = \theta(t)$.

Lagrange Equations: Derivation Outline for Systems of Particles

A little arithmetic shows that

$$\frac{\partial}{\partial t}\vec{e}_r(\theta) = \dot{\theta}\vec{e}_\theta(\theta) \textbf{ and that } \frac{\partial}{\partial t}\vec{e}_\theta(\theta) = -\dot{\theta}\vec{e}_r(\theta)$$

Recalling that $\vec{g} = r\vec{e}_r(\theta)$ **we derive**

$$\dot{\vec{g}} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta \textbf{ and } \ddot{\vec{g}} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + 2\dot{r}\dot{\theta}\vec{e}_\theta$$

Expressing any imposed force in this systems as $\vec{F} = F_r\vec{e}_r + F_\theta\vec{e}_\theta$ **we find that Newton's Law gives us**

$$F_r = \ddot{r} - r\dot{\theta}^2 \textbf{ and } F_\theta = 2\dot{r}\dot{\theta}$$

What do we observe from $F_r = \ddot{r} - r\dot{\theta}^2$ and $F_\theta = 2\dot{r}\dot{\theta}$?

- **Because this is not an orthogonal Cartesian coordinate system:**

  $F_r \neq \ddot{r}$ **and** $F_\theta \neq \ddot{\theta}$

- **The form of the scalar equations of motion depends on the coordinate system used**

- **We had to be smart to deduce the equations of motion**

Lagrange Equations: Derivation Outline for Systems of Particles

Lets find a method that does not require us to be so smart. One such method is the use of Lagrange Equations.

- **The derivation shown here is not that of the text, but it is the more standard approach.**
- **Students are not responsible for reproducing this derivation, but you will see it again in your studies of applied mechanics. It would be good to be able to follow the derivation.**
- **The derivation shown here is for systems of particles. The extension to systems of rigid bodies and of continua will be accepted on faith.**
- **The way to get comfortable with Lagrange equations is through practice.**

Lagrange Equations: Derivation Outline for Systems of Particles

We consider a system of N particles and set of n generalized coordinates $\{q_k\}$. It is assumed that these coordinates:

- **are enough to describe the configuration of the system of particles completely at any time, i.e.**

$$\vec{x}_k(t) = \vec{f}_k(q_1, q_2, \ldots q_n, t)$$

- **are independent, i.e.** $\mathrm{Rank}\left\{\frac{\partial}{\partial q_l}\vec{f}_k\right\} = n$

- **are appropriately differentiable**

(We sometimes simplify the notation: $\frac{\partial \vec{x}_k}{\partial q_r} = \frac{\partial}{\partial q_r}\vec{f}_k$**.)**

Lagrange Equations: Derivation Outline for Systems of Particles

We differentiate our kinematic relations to derive an expression for particle velocity:

$$\dot{\vec{x}}_k(t) = \sum_r \left(\frac{\partial}{\partial q_r} \vec{f}_k \right) \dot{q}_r + \frac{\partial}{\partial t} \vec{f}_k$$

The rates $\dot{q}_r$ are sometimes called speeds. Note that

$\frac{\partial}{\partial \dot{q}_r} \dot{\vec{x}}_k = \frac{\partial}{\partial q_r} \vec{f}_k$**, so we conclude that** $\frac{\partial}{\partial \dot{q}_r} \dot{\vec{x}}_k(t) = \frac{\partial}{\partial q_r} \vec{x}_k$**. We shall use this later.**

With our expression for particle velocity we can construct kinetic energy in terms of the generalized degrees of freedom.

$$T = \frac{1}{2} \sum_k m_k \left[\sum_r \left(\frac{\partial}{\partial q_r} \vec{x}_k \right) \dot{q}_r + \frac{\partial}{\partial t} \vec{x}_k \right] \cdot \left[\sum_s \left(\frac{\partial}{\partial q_s} \vec{x}_k \right) \dot{q}_s + \frac{\partial}{\partial t} \vec{x}_k \right]$$

Lagrange Equations: Derivation Outline for Systems of Particles

The partial derivative of kinetic energy on speed is called the generalized momentum:

$$p_r = \frac{\partial T}{\partial \dot{q}_r} = \sum_k m_k \left(\dot{\vec{x}}_k \cdot \frac{\partial}{\partial \dot{q}_r} \dot{\vec{x}}_k \right).$$

Because Newton's equations involve the rate of change of momentum, we are motivated to take the following time derivative:

$$\frac{dp_r}{dt} = \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_r}\right) = \sum_k m_k \left(\ddot{\vec{x}}_k \cdot \frac{\partial \dot{\vec{x}}_k}{\partial \dot{q}_r} \right) + \sum_k m_k \left[\dot{\vec{x}}_k \cdot \frac{d}{dt}\left(\frac{\partial \dot{\vec{x}}_k}{\partial \dot{q}_r} \right) \right].$$

We shall simplify each of the summations on the right hand side of the above equation.

Lagrange Equations: Derivation Outline for Systems of Particles

Simplifying $\sum_k m_k \left(\ddot{\vec{x}}_k \cdot \frac{\partial \dot{\vec{x}}_k}{\partial \dot{q}_r} \right)$

Recalling that $\frac{\partial \dot{\vec{x}}_k}{\partial \dot{q}_r} = \frac{\partial \vec{x}_k}{\partial q_r}$,

$$\sum_k m_k \left(\ddot{\vec{x}}_k \cdot \frac{\partial \dot{\vec{x}}_k}{\partial \dot{q}_r} \right) = \left(\sum_k m_k \ddot{\vec{x}}_k \cdot \frac{\partial \vec{x}_k}{\partial q_r} \right) = \sum_k \vec{F}_k \cdot \frac{\partial \vec{x}_k}{\partial q_r} = F_r$$

F_r **is called the Generalized Force associated with the generalized coordinate** q_r**.**

Lagrange Equations: Derivation Outline for Systems of Particles

Simplifying $\sum_k m_k \left[\dot{\vec{x}}_k \cdot \frac{d}{dt}\left(\frac{\partial \dot{\vec{x}}_k}{\partial \dot{q}_r} \right) \right] = \sum_k m_k \left[\dot{\vec{x}}_k \cdot \frac{d}{dt}\left(\frac{\partial \vec{x}_k}{\partial q_r} \right) \right]$,

We note that the derivative in the summation is:

$$\frac{d}{dt}\left(\frac{\partial \vec{x}_k}{\partial q_r} \right) = \sum_s \frac{\partial^2 \vec{x}_k}{\partial q_r \partial q_s} \dot{q}_s + \sum_s \frac{\partial^2 \vec{x}_k}{\partial q_r \partial t}$$

$$= \frac{\partial}{\partial q_r} \left[\sum_s \frac{\partial \vec{x}_k}{\partial q_s} \dot{q}_s + \sum_s \frac{\partial \vec{x}_k}{\partial t} \right] = \frac{\partial}{\partial q_r} \vec{x}_k$$

and obtain

$$\sum_k m_k \left[\dot{\vec{x}}_k \cdot \frac{d}{dt}\left(\frac{\partial \vec{x}_k}{\partial q_r} \right) \right] = \sum_k m_k \left[\dot{\vec{x}}_k \cdot \frac{\partial}{\partial q_r} \dot{\vec{x}}_k \right]$$

$$= \frac{\partial}{\partial q_r} \sum_k \frac{m_k}{2} [\dot{\vec{x}}_k \cdot \dot{\vec{x}}_k] = \frac{\partial T}{\partial q_r}$$

Lagrange Equations: Derivation Outline for Systems of Particles

We combine all the above to find that $\frac{dp_r}{dt} = F_r + \frac{\partial T}{\partial q_r}$.

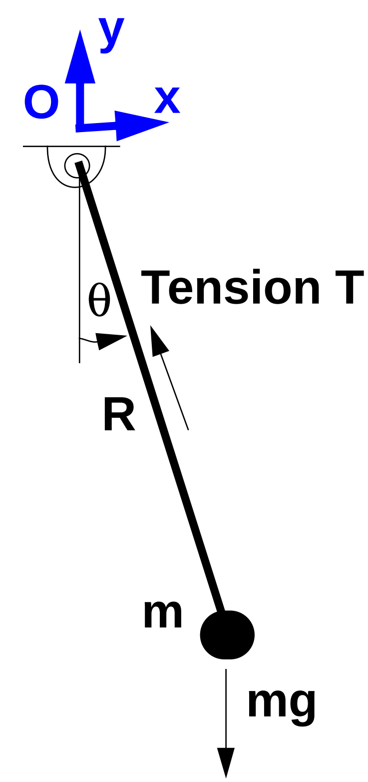
The more common form of this is $\boxed{\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_r}\right) - \frac{\partial T}{\partial q_r} = F_r}$

The above are Lagrange's equations.

- They hold in any system of coordinates.
- They reduce down to the component form of Newton's Law for orthogonal, Cartesian system.
- They are an invariant recipe.
- They accommodate constraints. This is shown later.

Lagrange Equations:
First Example Problem

Consider the pendulum shown, consisting of a mass suspended by a massless rod. The only degree of freedom is θ. Lets proceed with the steps of the recipe.



1. Express configuration in terms of the generalized coordinates: $\vec{x} = R\sin(\theta)\vec{i} - R\cos\theta\,\vec{j}$

2. Evaluate the Jacobian. These terms are really important. $\dfrac{\partial}{\partial\theta}\vec{x} = R\cos\theta\,\vec{i} + R\sin\theta\,\vec{j}$

3. Write the applied forces: $\vec{F} = -mg\vec{j} - T(\vec{x}/|\vec{x}|)$

4. Calculate generalized forces: $F_r = \vec{F}\cdot\dfrac{\partial}{\partial\theta}\vec{x} = -mgR\sin\theta$.

Lagrange Equations: First Example Problem

5. Calculate kinetic energy:

$$T = \frac{m}{2}(\dot{\vec{x}} \cdot \dot{\vec{x}}) = \frac{m}{2}\left(\frac{\partial}{\partial\theta}\vec{x} \cdot \frac{\partial}{\partial\theta}\vec{x}\right)\dot{\theta}^2 = \frac{m}{2}R^2\dot{\theta}^2$$

6. Plug and Chug $\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\theta}}\right) = mR^2\ddot{\theta} = -mgR\sin\theta$

Observe

- **We did not have to make any smart observations to evaluate the governing equation**
- **The tension on the rod did not play any role in the equation for $\ddot{\theta}$. This is because forces of constraint do not act in the directions of motion, so they do no work. We shall see this some more later.**

Degrees of Freedom and Constraints: Several Approaches

1. Because we have freedom in choosing our generalized degrees of freedom, we can exploit the constraints to minimize the number of degrees of freedom.

2. Where our constraints are holonomic (of the sort $h(q_1, q_2, \ldots q_n) = 0$) we can use the constraints to resolve out extra degrees of freedom before calculating kinetic energy or generalized forces.

We shall practice these later.

Degrees of Freedom
Potential Energies

Sometimes, we can simplify the calculations of generalized forces when they derive from a potential energy. At the moment, we can identify several sorts of force:

conservative external force - such as gravity

conservative internal force - such as strain energy

non-conservative external force - generally the case

non-conservative internal force - damping elements

Degrees of Freedom
Potential Energies

Potential energies give rise to conservative forces. The forces depend only on current configuration and can be written in terms of gradients of the corresponding potential energy.

$$F = -\nabla V = -\frac{\partial}{\partial \vec{x}} V$$

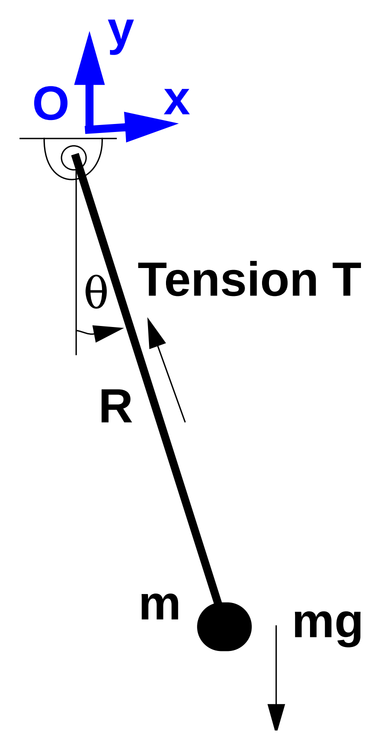
Note the nice feature of the chain rule for conservative forces:

$$F_r = F \cdot \frac{\partial}{\partial q_r} \vec{x} = -\frac{\partial V}{\partial \vec{x}} \cdot \frac{\partial}{\partial q_r} \vec{x} = -\frac{\partial V}{\partial q_r}$$

This facilitates the formulation of the equations.

Lagrange Equations: First Example Problem Revisited

Consider the pendulum shown, consisting of a mass suspended by a massless rod. The only degree of freedom is θ. Lets proceed with the steps of the receipt.



1. Again, express the configuration in terms of generalized coordinates:

$$\vec{x} = R\sin(\theta)\vec{i} - R\cos\theta\,\vec{j}$$

2. Evaluate the Jacobian. $\dfrac{\partial}{\partial\theta}\vec{x} = R\cos\theta\,\vec{i} + R\sin\theta\,\vec{j}$

3. There are only conservative forces $\vec{F} = -\nabla(mgy)$ **from which we deduce**

$$F_\theta = -\frac{\partial}{\partial\theta}mgy = -\frac{\partial}{\partial\theta}(-mgR\cos\theta) = -mgR\sin\theta$$

4. Calculate kinetic energy:

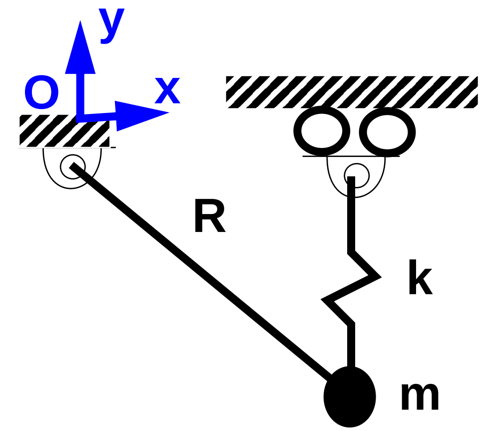
$$T = \frac{m}{2}(\dot{\vec{x}} \cdot \dot{\vec{x}}) = \frac{m}{2}\left(\frac{\partial}{\partial\theta}\vec{x} \cdot \frac{\partial}{\partial\theta}\vec{x}\right)\dot{\theta}^2 = \frac{m}{2}R^2\dot{\theta}^2$$

5. Plug and Chug $\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\theta}}\right) = mR^2\ddot{\theta} = -mgR\sin\theta$

Note that by using the potential energy, we were able to ignore irrelevant forces (the tension in the rod).

Another Example

Consider the following system involving both spring tension and gravity.



1. Again, the location of the mass is expressed in terms of the generalized coordinate θ,

$$\vec{x} = R\sin(\theta)\vec{i} - R\cos\theta\vec{j}$$

2. Evaluate the Jacobian

3. The generalized forces are calculated. This time we only need to consider potential energies: $V = mgy + ky^2/2$.

We use our recipe for generalized force:

$$F_\theta = -\frac{\partial V}{\partial \theta} = -\frac{\partial}{\partial \theta}[-mgR\cos\theta + kR^2\cos\theta^2/2]$$

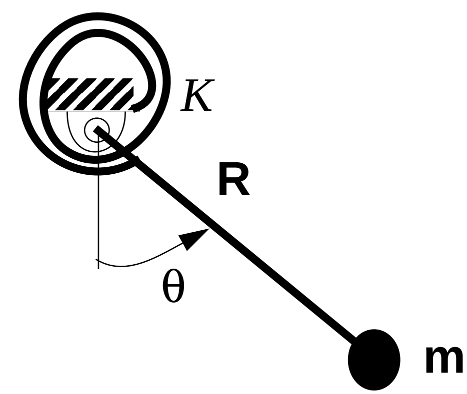
$$= -mgR\sin(\theta) + kR^2\sin(2\theta)/2$$

4. Again, the kinetic energy is $T = m(R^2\dot{\theta}^2)/2$

5. Plug and chug: $mR^2\ddot{\theta} = -mgR\sin\theta + kR^2\sin(2\theta)/2$

Lets do and EASY one

Consider the following system involving both a torsional spring tension and gravity.



1. Again, the location of the mass is expressed in terms of the generalized coordinate θ,

$$\vec{x} = R\sin(\theta)\vec{i} - R\cos\theta\vec{j}.$$

2. Evaluate the Jacobian

3. The generalized forces are calculated. This time we only need to consider potential energies to do this: $V = mgy + \kappa\theta^2/2$ **.**
We use our recipe for generalized force:

$$F_\theta = -\frac{\partial V}{\partial \theta} = -\frac{\partial}{\partial \theta}[-mgR\cos\theta + \kappa\theta^2/2]$$

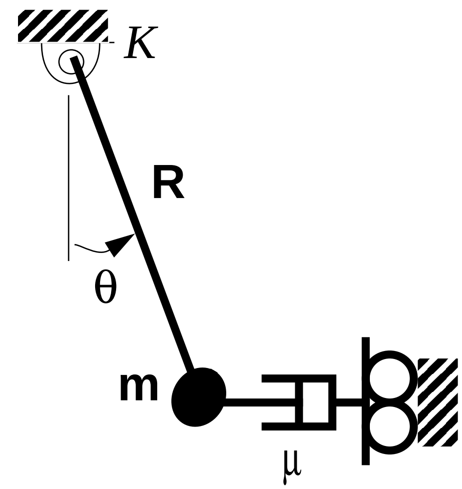
$$= -mgR\sin(\theta) - \kappa\theta$$

4. Again, the kinetic energy is $T = m(R^2\dot{\theta}^2)/2$

5. Plug and chug: $mR^2\ddot{\theta} = -mgR\sin\theta - \kappa\theta$

Lets Do One with Damping

Here we consider a another simple problem. It contains both conservative and dissipative forces:



1. Again, the location of the mass is expressed in terms of the generalized coordinate θ,

$$\vec{x} = R\sin(\theta)\vec{i} - R\cos\theta\vec{j}$$

2. Evaluate the Jacobian.

3. The generalized forces are calculated. This time we to consider potential energy as well as a force due to viscous damping and add the forces: $F_\theta = F_\theta^C + F_\theta^D$. As before, the force due to the potential energy $V = mgy$ is

$$F_\theta^C = -\frac{\partial V}{\partial \theta} = -\frac{\partial}{\partial \theta}[-mgR\cos\theta]$$
$$= -mgR\sin(\theta)$$

The force due to damping must be calculated in the brute force manner:

$$F_\theta^D = \vec{F}^D \cdot \frac{\partial}{\partial \theta}\vec{x} = -\mu\dot{x}\vec{i} \cdot \frac{\partial}{\partial \theta}\vec{x}$$
$$= -(\mu\dot{\theta}R[\cos\theta\vec{i}] \cdot R[\cos\theta\vec{i} + \sin\theta\vec{j}])$$
$$= -\mu\dot{\theta}R^2\cos\theta^2$$

4. Again, the kinetic energy is $T = m(R^2\dot{\theta}^2)/2$

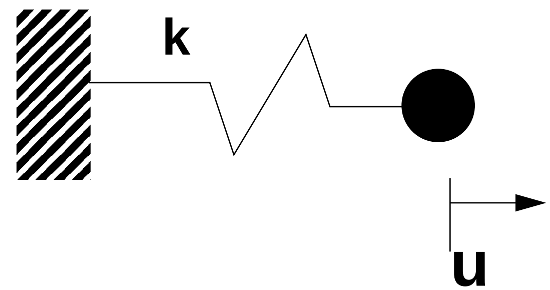
5. Plug and chug: $mR^2\ddot{\theta} = -mgR\sin\theta - \mu\dot{\theta}R^2\cos\theta^2$

That Was Hard

There must be an easier way to handle viscous dissipation. That will be covered in the next lesson. We shall address more of these dissipative problems then.

Some Easy Conservative Problems
The simplest problem

The potential energy is trivially found:



$V = \frac{1}{2}ku^2$ **. The generalized force is**

$$F_u = -\frac{\partial V}{\partial u} = -ku.$$

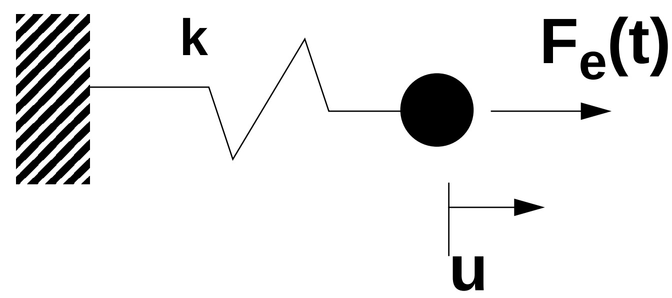
The kinetic energy is just as easily found: $T = \frac{m}{2}\dot{u}^2$ **, so**

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{u}}\right) - \frac{\partial T}{\partial u} = m\ddot{u} = -ku$$

That was easy. Lets do another.

Some Easy Non-Conservative Problems
The simplest problem with a force.

The potential energy is trivially found:



$V = \frac{1}{2}ku^2$ **. The generalized force is**

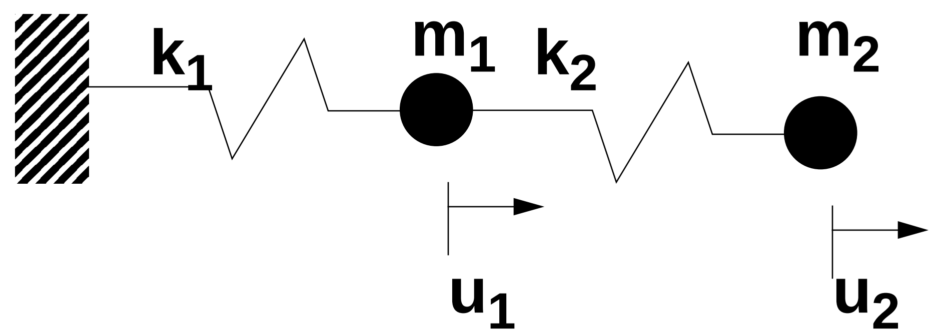
$$F_u = -\frac{\partial V}{\partial u} + F_e = -ku + F_e(t).$$

The kinetic energy is just as easily found: $T = \frac{m}{2}\dot{u}^2$ **, so**

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{u}}\right) - \frac{\partial T}{\partial u} = m\ddot{u} = -ku + F_e(t)$$

That was not so difficult. Lets do another just slightly more difficult.

Some Easy Non-Conservative Problems
A still-simple problem.



The potential energy is easily found: $V = \frac{1}{2}k_1 u_1^2 + \frac{1}{2}k_2(u_2 - u_1)^2$.

The generalized forces are

$$F_{u_1} = -\frac{\partial V}{\partial u_1} = -k_1 u_1 - k_2(u_1 - u_2)$$ and

$$F_{u_2} = -\frac{\partial V}{\partial u_2} = -k_2(u_2 - u_1).$$

The kinetic energy is just as easily found: $T = \frac{m_1}{2}\dot{u}_1^2 + \frac{m_2}{2}\dot{u}_2^2$**, so**

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{u}_1}\right) - \frac{\partial T}{\partial u_1} = m_1 \ddot{u}_1 = -k_1 u_1 - k_2(u_1 - u_2)$$ **and**

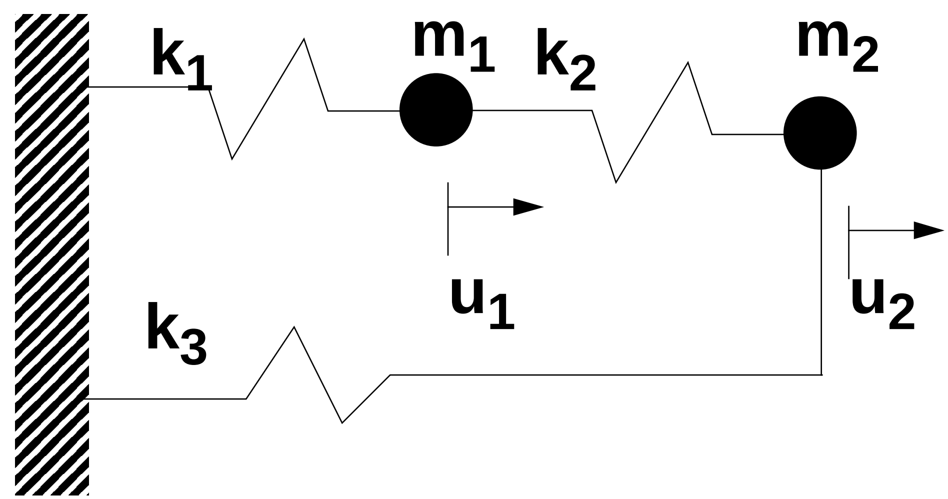
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{u}_2}\right) - \frac{\partial T}{\partial u_2} = m_2 \ddot{u}_2 = -k_2(u_2 - u_1)$$

In matrix form, $$\begin{bmatrix} m_1 & \\ & m_2 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Thus are developed mass and stiffness matrices

Lets do it again.

Some Easy Non-Conservative Problems
A still-simple problem.



The potential energy is easily found:

$$V = \frac{1}{2}k_1u_1^2 + \frac{1}{2}k_2(u_2 - u_1)^2 + k_3u_2^2.$$ **The generalized forces**

are $F_{u_1} = -\frac{\partial V}{\partial u_1} = -k_1u_1 - k_2(u_1 - u_2)$ **and**

$$F_{u_2} = -\frac{\partial V}{\partial u_2} = -k_2(u_2 - u_1) - k_3u_2.$$

The kinetic energy is just as still: $T = \frac{m_1}{2}\dot{u}_1^2 + \frac{m_2}{2}\dot{u}_2^2$**, so**

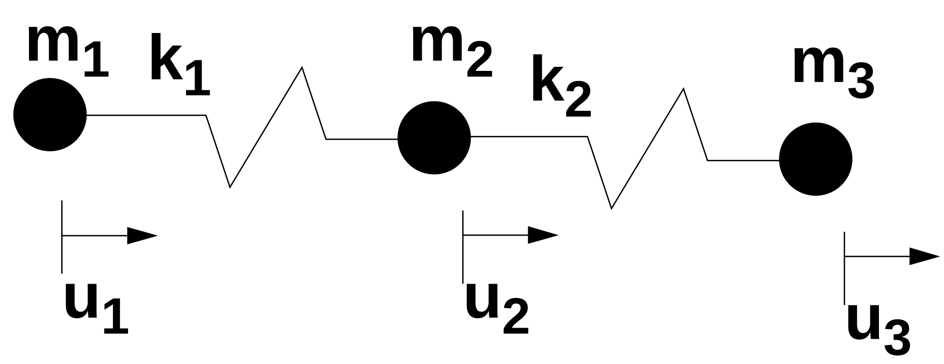
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{u}_1}\right) - \frac{\partial T}{\partial u_1} = m_1 \ddot{u}_1 = -k_1 u_1 - k_2(u_1 - u_2)$$ **and**

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{u}_2}\right) - \frac{\partial T}{\partial u_2} = m_2 \ddot{u}_2 = -k_2(u_2 - u_1) - k_3 u_2$$

In matrix form, $$\begin{bmatrix} m_1 & \\ & m_2 \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This is still easy.

Lets do a funny one



The potential energy is again easily found:

$$V = \frac{1}{2}k_1(u_2 - u_1)^2 + \frac{1}{2}k_2(u_3 - u_2)^2$$ **. The generalized forces are:**

$$F_{u_1} = -\frac{\partial V}{\partial u_1} = -k_1(u_1 - u_2)$$

$$F_{u_2} = -\frac{\partial V}{\partial u_2} = -k_1(u_2 - u_1) - k_2(u_2 - u_3)$$

$$F_{u_3} = -\frac{\partial V}{\partial u_3} = -k_2(u_3 - u_2)$$

The kinetic energy is: $T = \frac{m_1}{2}\dot{u}_1^2 + \frac{m_2}{2}\dot{u}_2^2 + \frac{m_3}{2}\dot{u}_3^2$.

The Lagrange equations now give the matrix form

$$\begin{bmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \end{bmatrix} + \begin{bmatrix} k_1 & -k_1 & \\ -k_1 & k_1 + k_2 & -k_2 \\ & -k_2 & k_2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

This is still easy.

About Mass and Stiffness Matrices

What can we conclude from our limited experience?

- **the mass matrix is symmetric**
- **the mass matrix is nonsingular (so long as each d.o.f. has mass)**
- **the mass matrix is positive definite**
- **the stiffness matrix is symmetric**
- **the stiffness is positive, semi-definite (rigid body motions).**

A Summary of the Recipe

1. Express the cartesian coordinates in terms of the generalized d.o.f.s

2. Calculate the partial derivatives of position w.r.t. generalized d.o.f.s

3. Express the potential energies in terms of the generalized d.o.f.s

4. Evaluate the generalized forces in terms of the generalized d.o.f.s

5. Evaluate the kinetic energy in terms of the generalized d.o.f.s

6. Substitute into the famous $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_r}\right) - \frac{\partial T}{\partial q_r} = F_r$

Lecture 5, Homework 1

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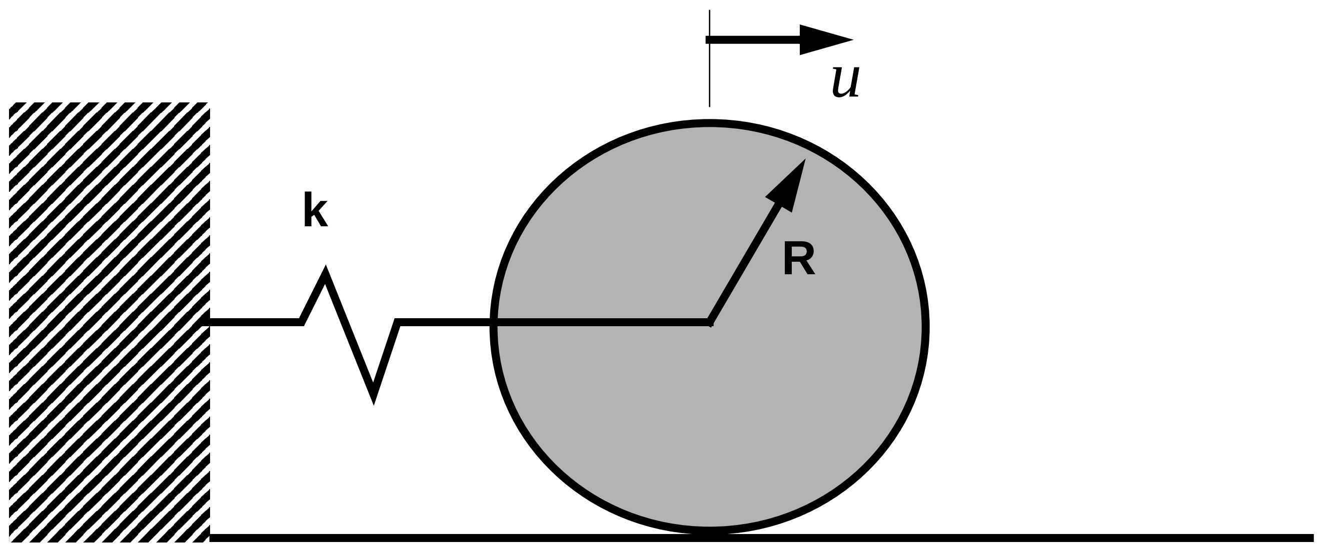
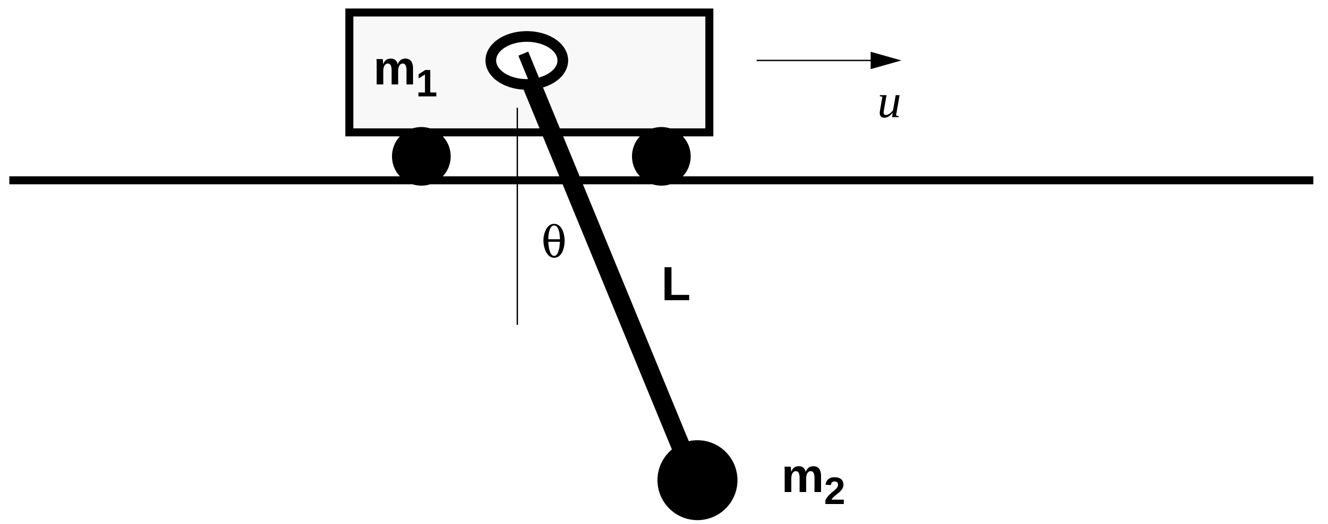


The disk shown has radius R, mass m, and mass moment of inertia J. The disk rolls without slipping and is restrained by a spring of stiffness k. Use the method of Lagarange's equations to derive an equation of motion for the displacment, u, of the center of gravity of the disk.

Hint: $R\dot{\theta} = \dot{u}$.

Lecture 5, Homework 2

Derive the equations of motion for the following 2-d.o.f. problem:



Next Time

We shall compare solution to homework 2B.

We shall go over today's problems again.

The Time after Next Time

Small Displacements and Linearization

Dissipation Potential

More on Mass and Stiffness Matrices