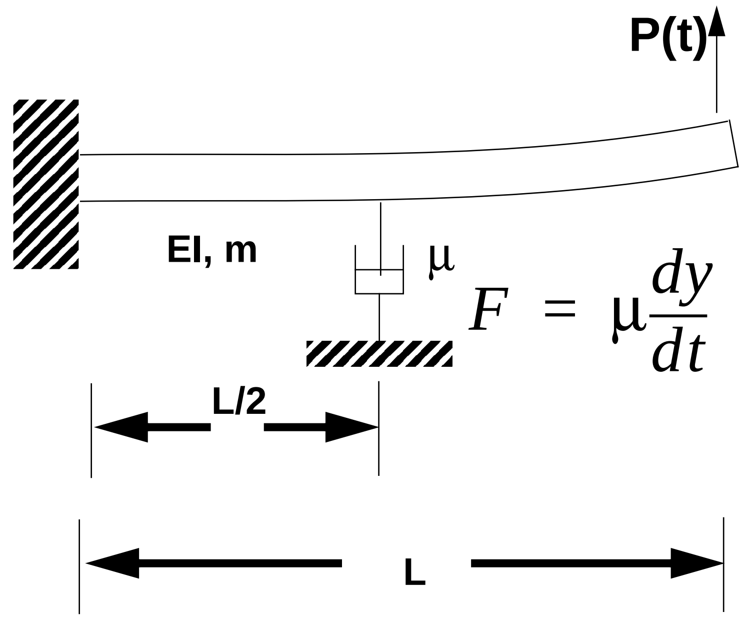
Mid-Term Take Home Exam
Problem 1. Assumed Modes

Consider the cantilevered Beam shown here. There is a time-dependent load applied at the end and a dashpot placed at the center of the span.



Assume a single deformation mode:

$$y(x, t) = A_1(t)\left(\frac{x}{L}\right)^2 .$$

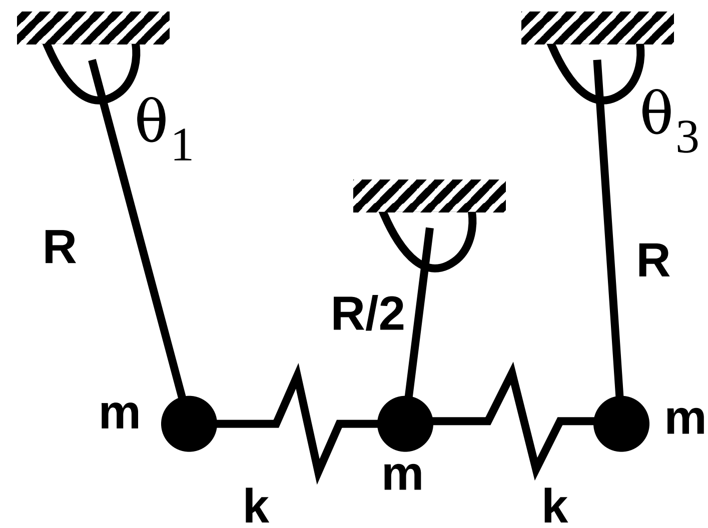
- **Derive the governing equation for $A_1(t)$.**

- **Assuming that P is periodic, calculate the *complex* magnification factor for the tip displacement as a function of the frequency.**

- **Calculate the time-averaged rate of energy dissipation as a function of frequency and force amplitude.**

Problem 2.

Consider the three pendula shown.



- Calculate the governing equations for the three angles θ_1, θ_2, & θ_3 in terms of the parameters shown. (Ignore vertical components of spring extension.)
- Linearize the equations, evaluating the mass and stiffness matrices.
- Impose the constraint that $\theta_1 = \theta_3$, writing the constraint matrix. Write the mass and stiffness matrices in the reduced system.
- Calculate the eigenmodes and frequencies in the reduced system.
- Express those eigenmodes in terms of the full set of displacement degrees of freedom, θ_1, θ_2, & θ_3.