# Advanced Vibrations



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# Advanced Vibrations
# Overview

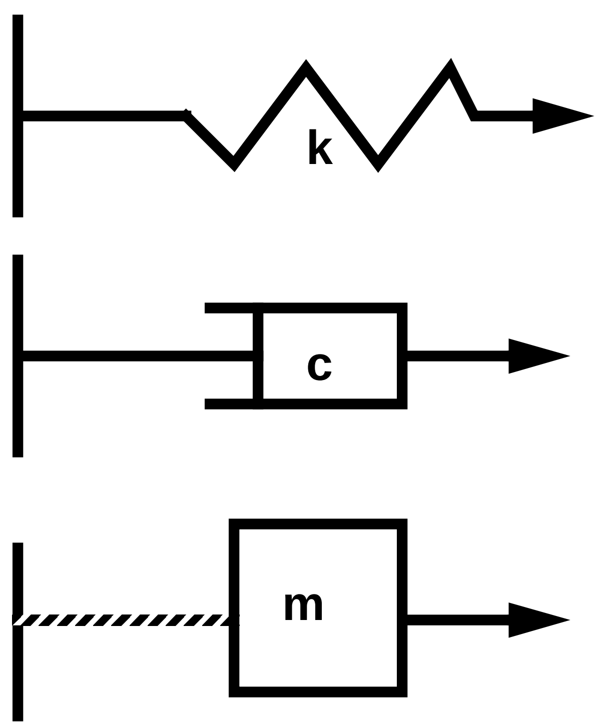


## Core Elements

- **Frequency Domain Analysis**
- **Energy Methods**
- **Continuous Systems**
- **Systems of Many D.O.F.s & Eigenanalysis**

# Lumped Mass Single DOF System Elements:





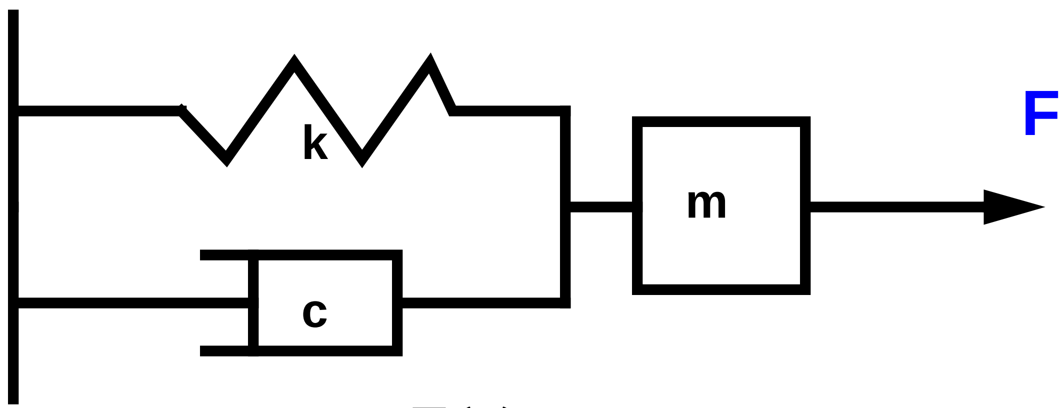

**Spring:** $F = kx$

**Damper:** $F = c\dot{x}$

**Mass:** $F = m\ddot{x}$

# Lumped Mass Single DOF System Assembled System:






**Applied force $F(t)$ is balanced by acceleration of mass, stroke of damper, and displacment of spring:**

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

# Lumped Mass Single DOF System Simplification by Mass Normalization



**Dividing through by the mass (we shall do this again later with multi-dof systems):**

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = f(t)$$

**where**

$$\frac{c}{m} = 2\zeta, \frac{k}{m} = \omega_n^2, \text{ and } \frac{F(t)}{m} = f(t)$$

**Recall:** $\omega_n$ **is the undamped natural frequency;**

**and** $\zeta$ **is fraction of critical damping. (Note:** $\zeta$ **is pronounced "zeta".)**

# Lumped Mass Single DOF System
# About the Differential Operator



**Note that the operator** $L = \dfrac{d^2}{dt^2}(\ \ ) + 2\zeta\omega_n\dfrac{d}{dt}(\ \ ) + \omega_n^2(\ \ )$ **has the properties:**

$L$ **is Linear:** $L(f+g) = L(f) + L(g)$

**and** $L$ **is time-translation invarient:** $L(x(t+T)) = L(x)\big|_{t+T}$

**This means that transform methods work for this sort of problem. We shall discuss this more next time.**

# Lumped Mass Single DOF System
# An Additional Special Normalization



**Nondimensionalization of equation for single DOF system:**

**Let** $\tau = \omega_n t$ **and let** $y(\tau) = x(\tau/\omega_n)$**, then**

$$\frac{dy}{d\tau} = \frac{1}{\omega_n^2}\frac{dx}{dt}\bigg|_{t = \tau/\omega_n}, \quad \frac{d^2 y}{d\tau^2} = \frac{1}{\omega_n^2}\frac{d^2 x}{dt^2}\bigg|_{t = \tau/\omega_n}$$

**and** $y''(\tau) + 2\zeta y'(\tau) + y(\tau) = \dfrac{f(\tau/\omega_n)}{\omega_n^2} = \dfrac{F(\tau/\omega_n)}{k}$

**Say** $F(t) = Ak\sin(\omega t)$**, then**

$$y''(\tau) + 2\zeta y'(\tau) + y(\tau) = A\sin\left(\tau\frac{\omega}{\omega_n}\right)$$

# Lumped Mass Single DOF System Nondimensionalization



**Consider the steady state solution of**

$$y''(\tau) + 2\zeta y'(\tau) + y(\tau) = \sin(\tau\alpha)$$

**where $\alpha = \omega/\omega_n$, we anticipate a solution of the sort**

$$y(\tau) = \tilde{H}(\alpha)\sin(\tau\alpha - \tilde{\psi}(\alpha))$$

**If we solve for $\tilde{H}(\alpha)$ and $\tilde{\psi}(\alpha)$ for all $\alpha$ for a fixed $\zeta$ we have the steady state solution for all single DOF systems with that value of $\zeta$.**

# Please read the first chapter of Meirovitch

**Life will be easier for you to read one chapter ahead.**

# Homework Number 1



**1. Solve the transient Single DOF problem**

$y''(\tau) + 2\zeta y'(\tau) + y(\tau) = \sin(\tau\alpha)$ **numerically using MATLAB until steady state for values of $\alpha$ equal to 0 to 2.0 in increments of 0.1.**

**2. Plot $\tilde{H}(\alpha)$ versus $\alpha$**

**3. plot $\tilde{\psi}(\alpha)$ versus $\alpha$**

**4. Do this for the three values of $\zeta$: 0.01, 0.1, 1.0**

**Hint: Let** $v = \begin{bmatrix} y' \\ y \end{bmatrix}$ **and solve** $v' + \begin{bmatrix} 2\zeta & 1 \\ -1 & 0 \end{bmatrix} v = \begin{bmatrix} \sin(\alpha\tau) \\ 0 \end{bmatrix}$