Slides of Lecture 6

Today's Class:

- **Solution to Homework 1 of Lecture 5.**
- **Short Quiz**
- **Comparison of answers to Homework 2B**
- **Practice with Lagrange Equations**

Lecture 5, Homework 1

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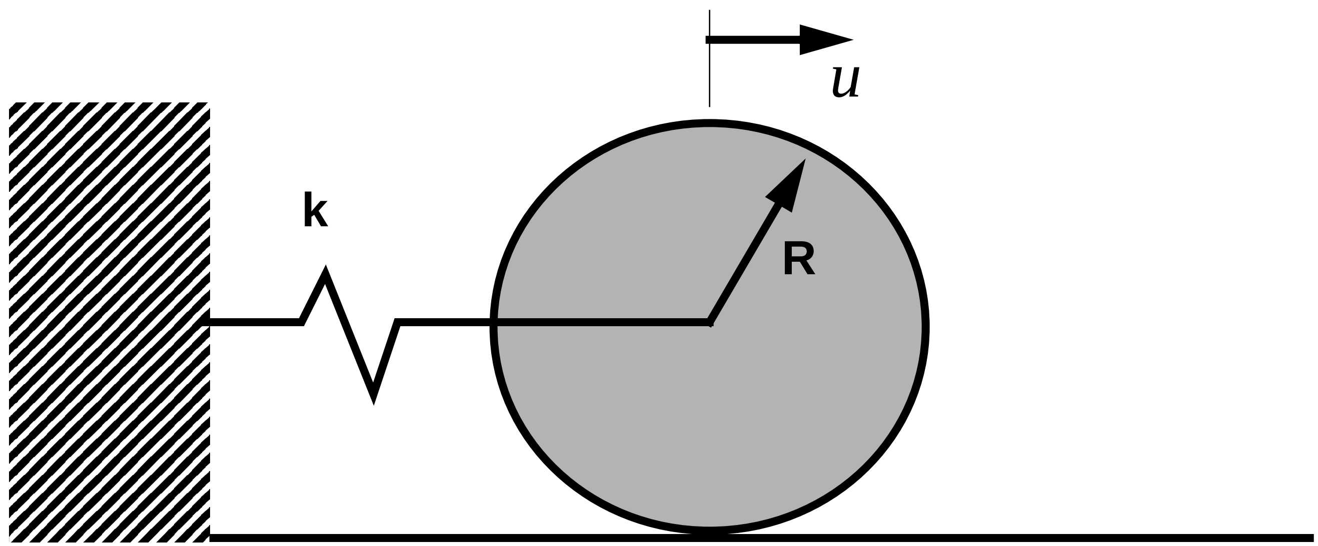


The disk shown has radius R, mass m, and mass moment of inertia J. The disk rolls without slipping and is restrained by a spring of stiffness k. Use the method of Lagrange's equations to derive an equation of motion for the displacement, u, of the center of gravity of the disk.

Hint: $R\dot{\theta} = \dot{u}$.

Lecture 5, Homework 1: Solution

Kinetic Energy: $T = \frac{1}{2}m\dot{u}^2 + \frac{1}{2}J\dot{\theta}^2$

Substituting $R\dot{\theta} = \dot{u}$ **into the expression for kinetic energy and performing the appropriate derivatives:**

$$\frac{d}{dt}\left(\frac{\partial T}{\partial u}\right) = (m + J/R^2)\ddot{u}$$

Potential Energy: $V = \frac{1}{2}ku^2$ **yields force:** $F_u = -\frac{\partial V}{\partial u} = -ku$

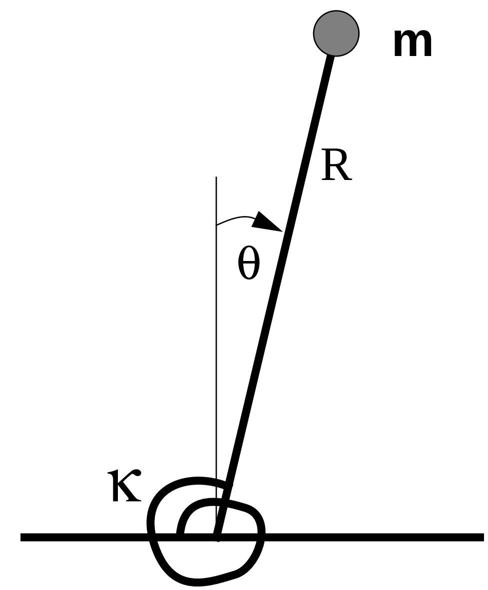
The potential energy yields a force $F_u = -\frac{\partial V}{\partial u} = -ku$

Putting it all together: $(m + J/R^2)\ddot{u} + ku = 0$

Short Quiz:

Consider the problem of an inverted pendulum. Derive the equation of motion of this structure in terms of the angle θ.

$$\vec{x} = R(\sin\theta\,\vec{i} + \cos\theta\,\vec{j})$$



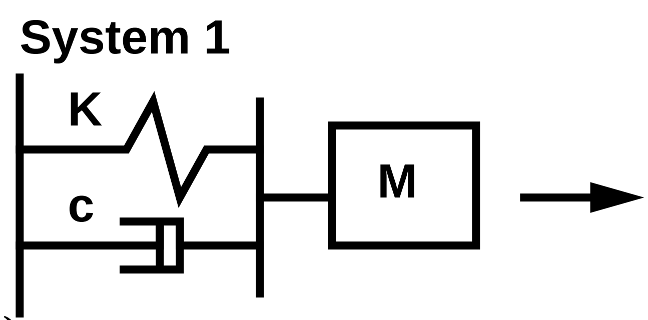
Note that the torsional spring serves to restore (center) the pendulum but that gravity serves to exacerbate any displacement from center.

Remember to write $T = \ldots$ and

$V = \ldots$

Resolution of Homework 2B

Determine the indicial response function $g(t)$ for System 1. Recall that



the indicial response function satisfies

$$u(t) = \int_0^t g(t-\tau)\dot{F}(\tau)d\tau + F(0)g(t)$$

We take the Laplace transform to find $Lu = (Lg)(sLF - F(0)) + (Lg)F(0) = (Lg)(sLF)$

Recall that the step response satisfies $Lu = (Lh)(LF)$ from which we conclude that $Lg = (Lh)/s$ and that

$$g(t) = \int_0^t h(\tau)d\tau$$

From an earlier homework, we found

$$h(t) = \frac{1}{M}\mathcal{L}^{-1}\left(\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right)$$

$$= \frac{e^{-\omega_n \zeta t}}{M\omega_n\sqrt{1-\zeta^2}}\sin(\omega_n\sqrt{1-\zeta^2}\,t)$$

We integrate this to find

$$g(t) = \int_0^t \frac{e^{-\omega_n \zeta \tau}}{M\omega_n\sqrt{1-\zeta^2}}\sin(\omega_n\sqrt{1-\zeta^2}\,\tau)\,d\tau$$

After a little integration, this becomes.

$$g(t) \; = $$

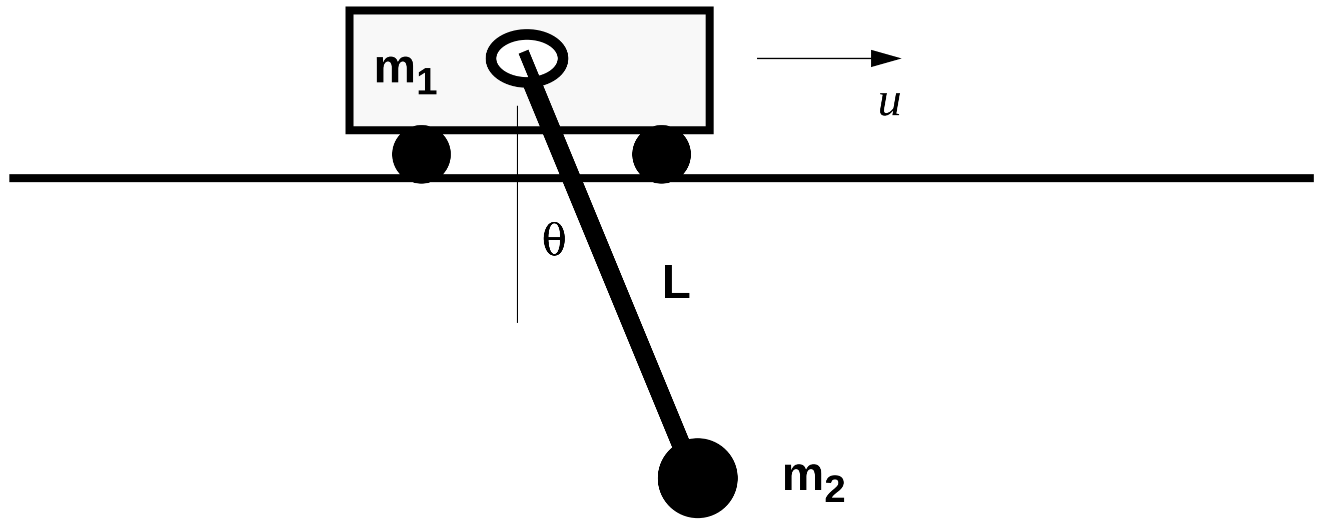
$$\frac{1 - e^{-\omega_n \zeta t}\left(\cos(\omega_n\sqrt{1-\zeta^2})t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\omega_n\sqrt{1-\zeta^2})t\right)}{K}$$

Simplifications have included the observations that

$(\omega_n \zeta)^2 + (\omega_n\sqrt{1-\zeta^2})^2 \; = \; \omega_n^2$ **and that** $M\omega_n^2 \; = \; K$

Remember to do the second homework given in Lecture 5.

Derive the equations of motion for the following 2-d.o.f. problem:



Express kinematics in terms of generalized degrees of freedom:

$\vec{x}_1 = u\vec{i}$ **and** $\vec{x}_2 = u\vec{i} + L\sin\theta\vec{i} - L\cos\theta\vec{j}$.

The potential energy is just that due to gravity: $V = -m_2 gL\cos\theta$

Next Time

- **Small Displacements and Linearization**
- **Dissipation Potential**
- **More on Mass and Stiffness Matrices**