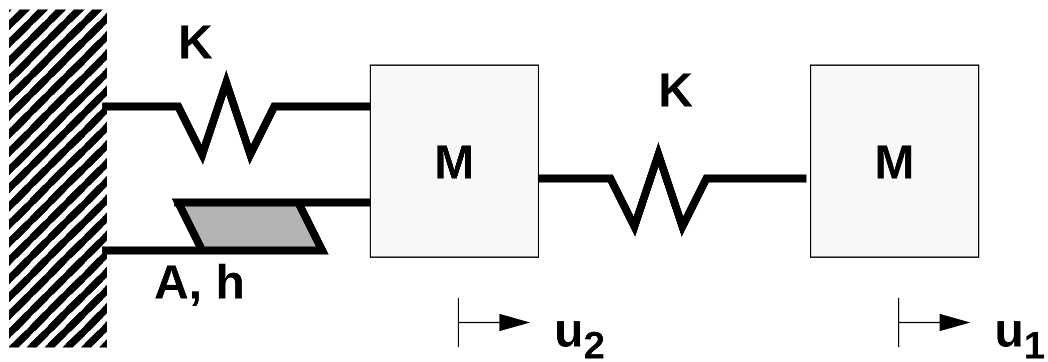
Problem E
A Damping Homework

We consider a structure such as that shown below



We have two masses, two springs, and a shear damper.The cross sectional area of the damper is $A$and the thickness of the damping material is h. The damping material is that discussed earlier. For

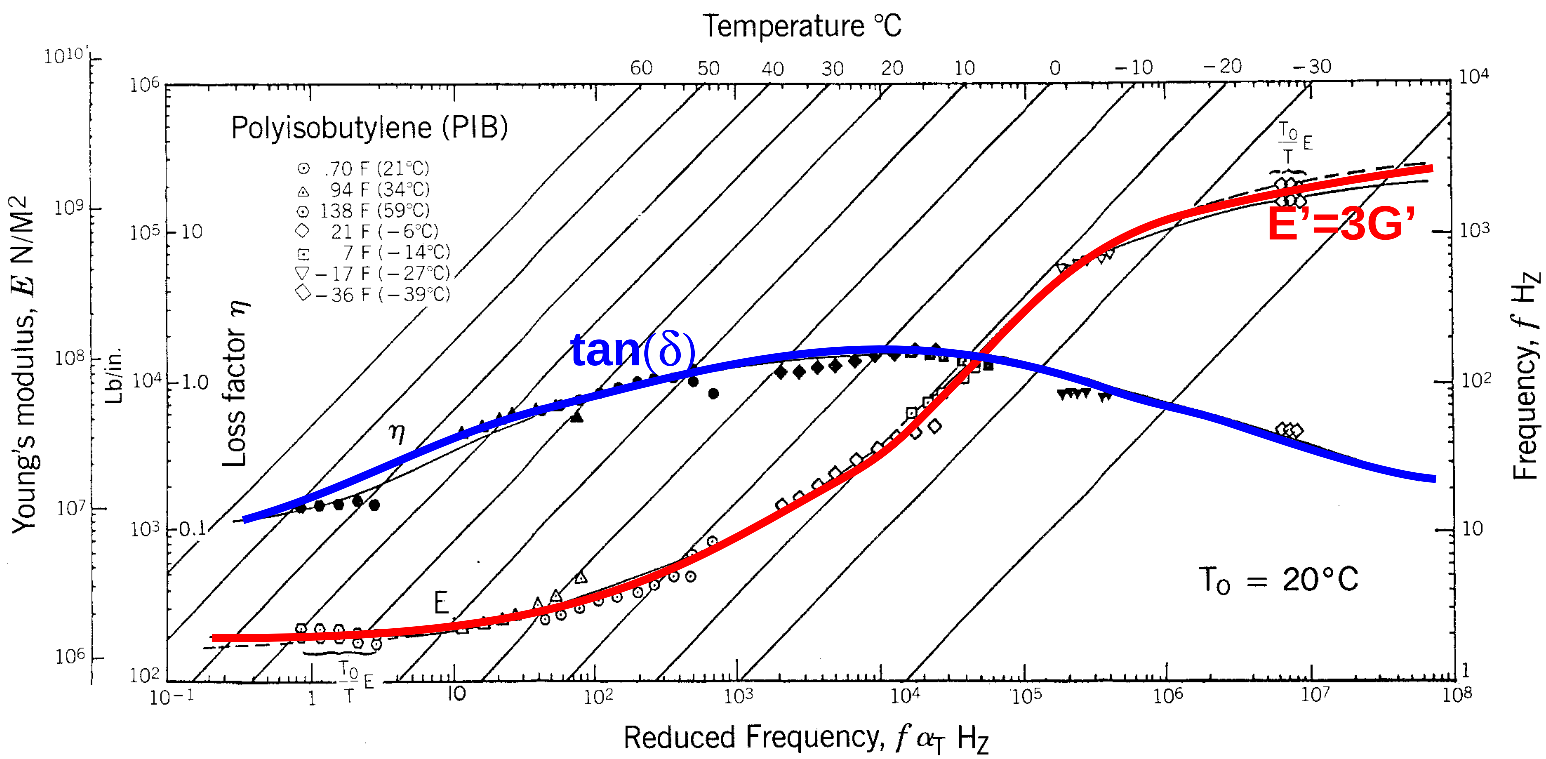
$M = 3$ **kg;** $K = 1.0\times10^6$ **N/m; and** $A = 0.01$ **m^2,**

1. **calculate the damping of both modes for $h = 0.01$ m**

2. **determine empirically the appropriate thickness, h, to make the small modal damping as large as possible.**

The Damping Material

The following plot for polyisobutylene is taken from “Vibration Damping” by Nashif et. al.



011C. Nomogram.

Interpolating Functions

We can fit the above plots reasonably well in $0.1 < \frac{\omega}{2\pi} < 10^5$ **by**

$$Log(3G'(\omega)) = Log(E'(\omega)) = 6.2 + 0.0112\left(Log\left(\frac{\omega}{2\pi}\right)\right)^{3.3}$$ **or**

$$G'(\omega) = 10^{5.72 + 0.0112\left(Log\left(\frac{\omega}{2\pi}\right)\right)^{3.3}}$$ **and**

$$Log(\tan\delta(\omega)) = -1 + 0.56Log\left(\frac{\omega}{2\pi}\right) - 0.069\left(Log\left(\frac{\omega}{2\pi}\right)\right)^2$$ **so**

$$G''(\omega) = G'(\omega)\tan\delta(\omega)$$

$$= 10^{4.72 + 0.56Log\left(\frac{\omega}{2\pi}\right) - 0.069\left(Log\left(\frac{\omega}{2\pi}\right)\right)^2 + 0.0112\left(Log\left(\frac{\omega}{2\pi}\right)\right)^{3.3}}$$

Solution
Using a lot of Matlab

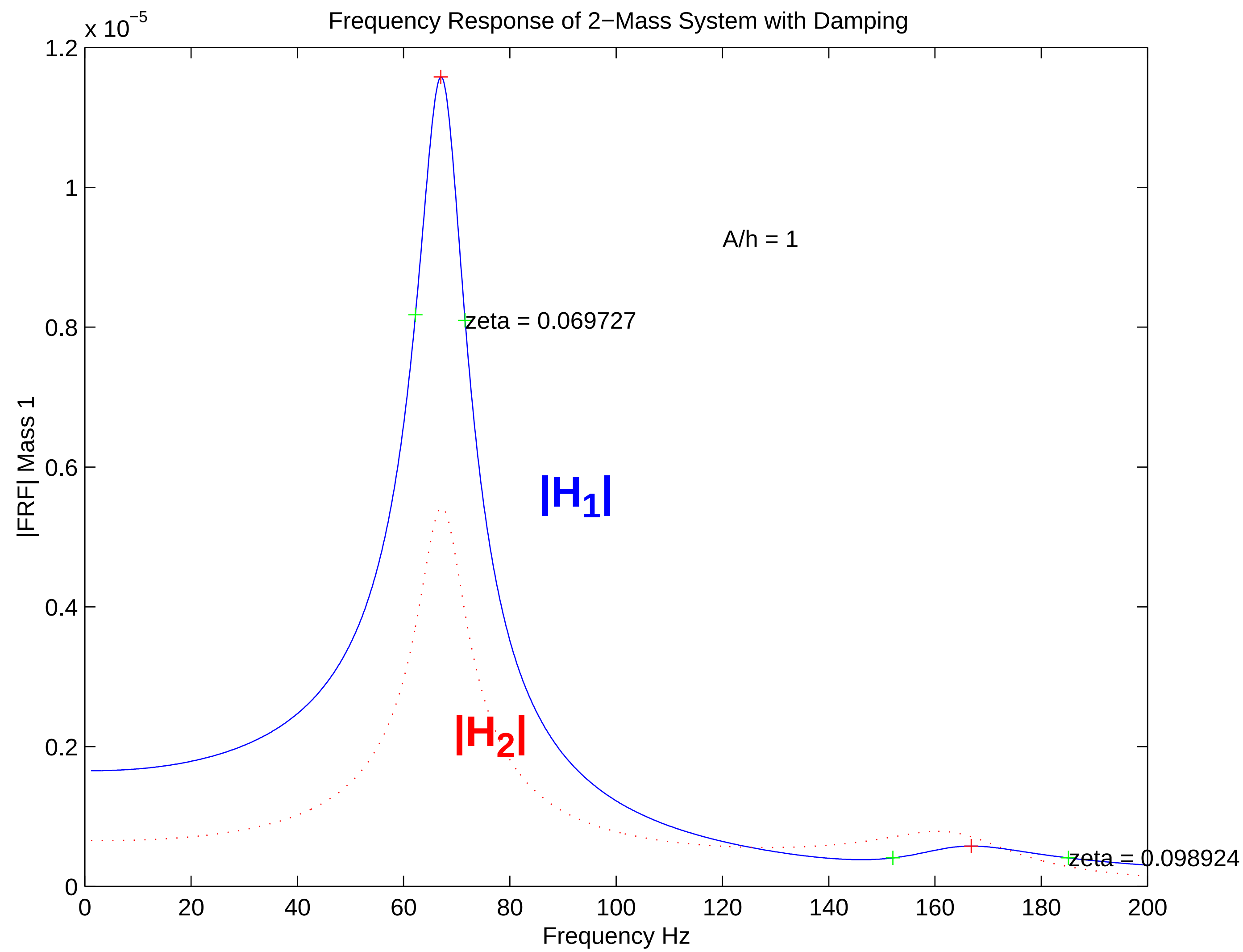
The governing equation in frequency space is

$$-\omega^2 \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} + \begin{bmatrix} K & -K \\ -K & (2K + G^*(\omega)(A/h)) \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} F(\omega) \\ 0 \end{bmatrix}$$

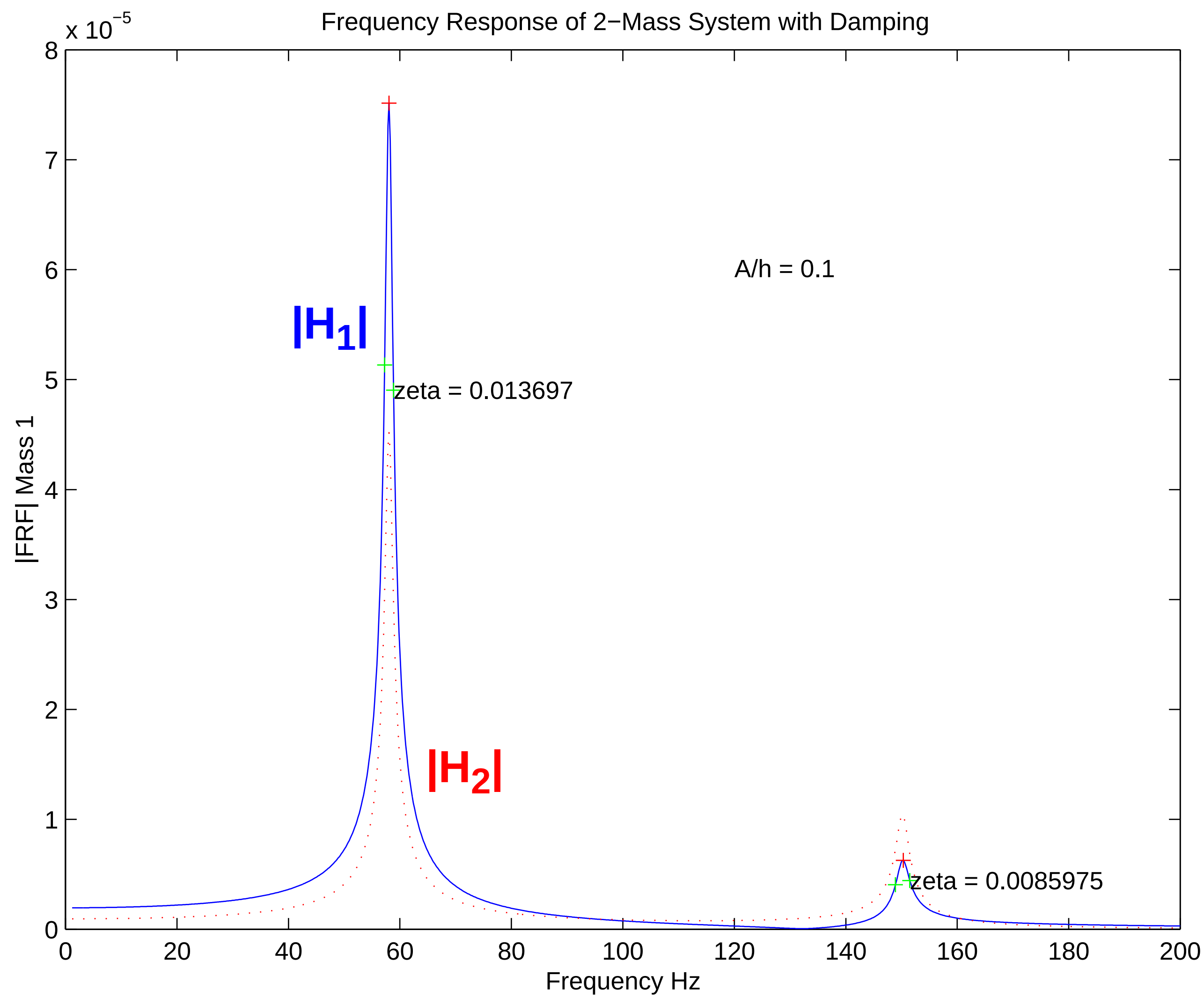
where we are mimicking a driving force on mass 1.

1. We employ matlab to invert the above system to solve for the driving point frequency response $H_1(\omega) = U_1(\omega)/F(\omega)$.
2. We plot $|H_1(\omega)|$ versus ω (or f, it makes no difference. why?)
3. Find the peaks
4. Determine fraction of critical damping for each mode via $\zeta = \dfrac{\Delta\omega}{2\omega_n}$

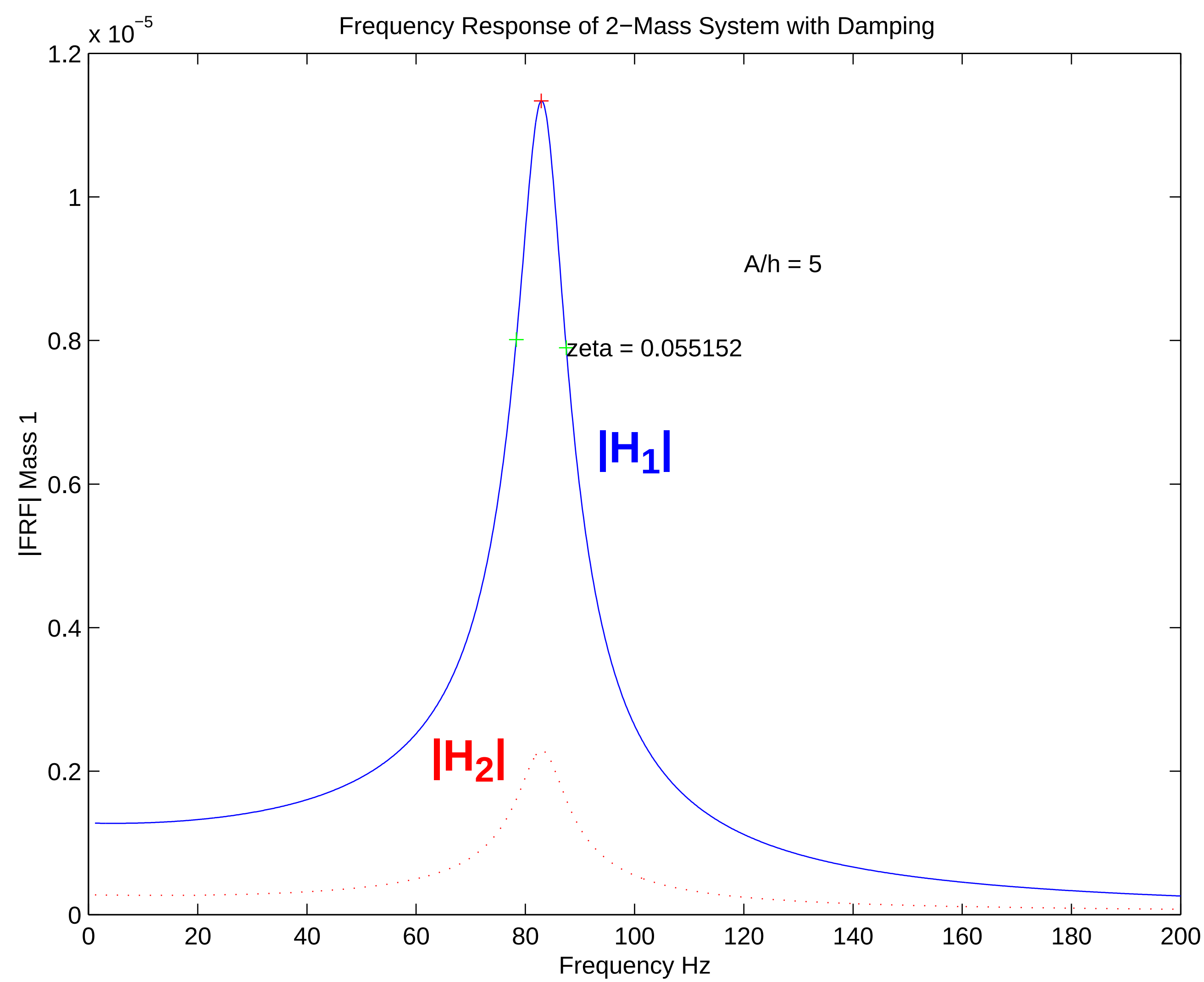
Results for $h = 0.01$ == $A/h = 1.0$



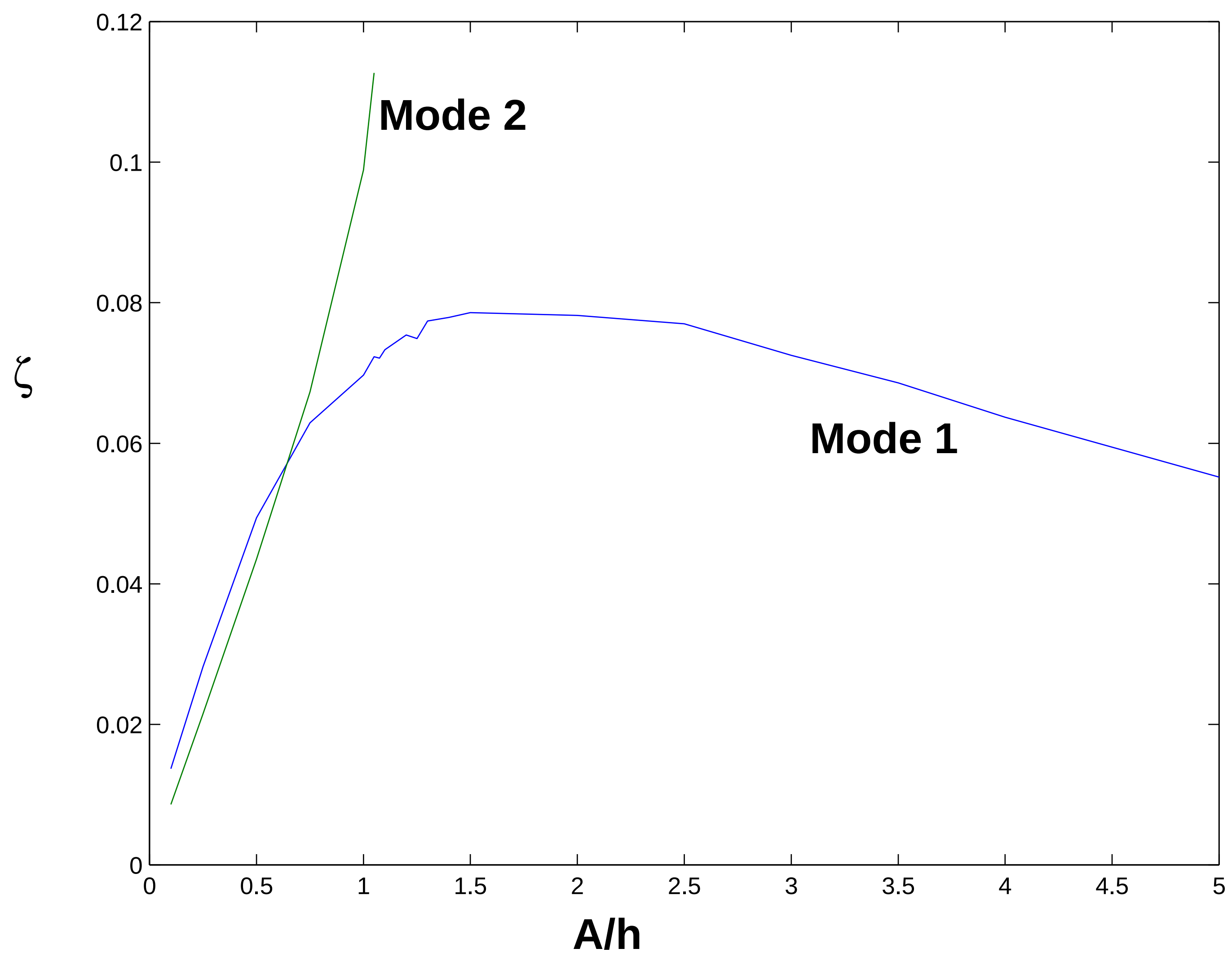
Results for $h = 0.1$ == $A/h = 0.1$



Results for $h = 0.002$ == $A/h = 5$



Results from many calculations



What do we observe from the above?

We note the following

- **Increasing thickness drops the resonant frequencies. Decreasing thickness raises natural frequencies.**
- **The second mode, which is associated with toward each other, is suppressed as the thickness is decreased. What is happening.**
- **The first mode, which is associated with masses moving in the same direction, does have a configuration of maximum damping.**
- **Optimal damping appears to happen in the vicinity of h = 0.005 m (5.0 mm)**

What do we imagine to the limiting mode as $h \rightarrow 0$?

Final Note

This would be only a marginally good candidate for the modal strain energy method.

The reason is that the viscoelastic represents an appreciable part of the stiffness of the structure. That means that the stiffnesses are appreciably different at different frequencies.