# Combining Cracks and Contact with Constitutive and Cohesive laws for Complete Calculations of Cutting

John A. Nairn<sup>1</sup> and Yamina Aimene<sup>2</sup>

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> MPM Workshop 14-15 March 2013, Salt Lake City, Utah

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# Motivation

- Cutting of solid wood and wood-based composites
  - Historically done by empirical methods (Koch, 1950s)
  - Almost no analysis for wood-based composites
    - Plywood
    - Particle Board
    - Oriented Strand Board



- Wood Plastic Composites
- Should be able to do better
  - *e.g.*, Atkins, Williams, *etc*.



"You can cut *Trex* just like regular wood."

# Experiments Too

- Build apparatus for cutting experiments
  - Based on Patel, Blackman, and Williams (from 5<sup>th</sup> ESIS TC4 meeting)
- New experiments and analysis
  - HDPE and LDPE
  - Trex (WPC)
  - Timber Tech (WPC)
  - Wood
- Theory and Numerical modeling
  - Material Point Method (MPM)



### Experiments

- Four materials HDPE, LDPE, Trex, and Timber Tech
- Rake Angles 15, 20, 22.5, 25, 30, 35, 40, 45, 50, and 55
- Depth of cut up 0.006 mm to 0.59 mm
- Semi-automatic data acquisition
- Most likely, the non-zero intercept relates to a "cutting toughness," but how it is best determined?







Griffith-Like Energy Balance (from Atkins)

$$F_c V = \tau_y \gamma(hbV) + T\left(\frac{V\sin\phi}{\cos(\phi - \alpha)}\right) + G_c bV$$

Work = Plastic Energy + Frictional Work + Fracture Work



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### Williams/Atkins Analysis — Chip Force Method

• Energy balance to forces, energy release rate, yield stress, and shear angle

$$\frac{\sigma_y}{2}\frac{h}{\sin\phi} = \left(\frac{F_c}{b} - G_c\right)\cos\phi - \left(\frac{F_t}{b} + G_c\tan\alpha\right)\sin\phi$$

• Observations show that  $T \neq \mu N$ , Williams tried:

$$T = G_a + \mu N \longrightarrow \frac{F_t}{b} = Z \frac{F_c}{b} + \frac{G_a}{\cos \alpha + \mu \sin \alpha}$$

Minimize work (to eliminate φ)

$$\frac{F_c}{b} = G_c + \sigma_y h \left( Z + \sqrt{1 + Z^2 + H} \right)$$
$$H = \frac{2}{\sigma_y h} \left( \frac{G_a}{\cos \alpha + \mu \sin \alpha} + G_c (Z + \tan \alpha) \right)$$

#### Sample Extrapolation



# "Multiphysics" Numerical Modeling







#### **Contact Physics**

- Tool = Rigid Material
- Coulomb Friction on chip and on bottom

#### MPM Contact

- a. Contact available for "free"
- b. But needs revision to work
- c. Key is contact normals (fixed in this problem)
- Simulation output total force on tool (same as F<sub>c</sub> and F<sub>t</sub> and new code option)



# "Multiphysics" Numerical Modeling



# MPM Contact

- Contact Detection
  - Volume Screening:  $V_{total} > V_c$
  - Approaching:  $\Delta \vec{p}_{i,a} \cdot \hat{n} < 0$
  - Overlap:  $\vec{\delta}_i \cdot n \delta_{contact} < 0$
- Normal Vector Calculation Options
  - MVG: maximum volume gradient
  - AVG: average volume gradient
  - SN: Specify the normal
- Extension of Contact to Model Interfaces

 $T_n = D_n[u_n]$  and  $T_t = D_t[u_t]$ 



#### One Imperfect Interface Result



#### Challenges

- 1. Finding contact area for arbitrary interface orientation
- 2. Working with stiff interfaces  $D_n, D_t \to \infty$

J.A. Nairn, "Modeling Imperfect Interfaces in the Material Point Method using Multimaterial Methods," Computer Modeling in Eng. & Sci., in press (2013) — <u>http://www.cof.orst.edu/cof/wse/faculty/Nairn/papers/MMInterfaces.pdf</u>

15°

30°

45°







#### Model Verification

- Elastic, perfectly-plastic material
  - E = 1000 MPa, v = 0.33,  $\sigma_y = 25$  MPa
  - Plane strain analysis
- Simple fracture law
  - n = 1,  $G_{lc} = G_{llc} = G_c = 2000 \text{ J/m}^2$  (constant  $G_c$  regardless of mode)
  - Cubic traction law,  $\sigma_c = 40$  MPa
- Frictionless contact
- Compare to analytical model
  - J. G. Williams, Eng. Fract. Mech., 77, 293-308 (2010).

# Simulations Problems

• Numerical difficultly resolving contact at the tool tip



"Elastic-plastic bending"





### Simulations vs. Plastic Bending Analysis



### Simulations vs. Plastic Bending Analysis



#### Simulations Reveal Non-Negligble Bottom Force





#### Semi-Analytical Model

- Revise elastic-plastic bending for  $F_b^n$
- Insert  $F_b^n$  from simulation results

$$T \neq G_a + \mu N$$
 but instead:  
 $T = \mu N$   
 $F_b^t = \mu F_b^n$   $\longrightarrow$   $\frac{F_t}{b} = Z \frac{F_c}{b} + (1 - \mu Z) \frac{F_b^n}{b}$ 

![](_page_27_Figure_1.jpeg)

#### Simulations/Modeling with Friction

![](_page_28_Figure_1.jpeg)

#### Effect of Cohesive Stress

![](_page_29_Figure_1.jpeg)

## Effect of Toughness

![](_page_30_Figure_1.jpeg)

#### Effect of Rake Angle

![](_page_31_Figure_1.jpeg)

### Hyperelastic-Plastic, Large Strain Material

Hyperelastic

Hypoelastic

**Cumulative Plastic Strain** 

### Hyperelastic-Plastic, Large Strain Material

![](_page_33_Figure_1.jpeg)

Equivalent Stress

# **Cutting Forces**

![](_page_34_Figure_1.jpeg)

### In-conclusions

- HDPE, LDPE, Trex, and Timber Tech Experiments
  - Works reasonable well, but answer depends on interpretation of the  $F_t$  vs.  $F_c$  intercept.
  - New results for Trex and Timber Tech Wood Plastic Composites
- Numerical simulations (by MPM) are working
  - Uncertain validation
  - Potential simulations (e.g., veneer peeling) may be useful
- Forces on bottom of tool
  - How to handle it?
  - Related to sharpness
  - Essential to theory and to modeling

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![](_page_36_Picture_11.jpeg)

## Carnot Cycle or an MPMgine™

- Ideal Gas as Hyperelastic Material
- Custom boundary conditions
- Trick is when to switch on returning
- Why not other materials
  - Carnot claimed general result

![](_page_37_Figure_6.jpeg)

## Carnot Cycle on Other Materials?

- First step is cooling on isothermal expansion
  - True in coupled conduction-elasticity
  - Small effect, usually neglected

![](_page_38_Figure_4.jpeg)

- Tungsten with MG EOS
- But plasticity always heats?
- Eliminate yielding

![](_page_38_Figure_8.jpeg)

#### Ideal Rubber Elastic Material

 In 1805, John Gough described a series of experiments on caoutchouc or Indian rubber:

"For the resin evidently grows warmer the further it is extended; and the edges of the lips possess a high degree of sensibility, which enables them to discover these changes with greater facility than other parts of the body."

- Mooney-Rivlin Hyperelastic Material
- "Ideal Rubber" from Flory

$$\left(\frac{\partial U}{\partial L}\right)_T = 0 \quad \text{therefore} \quad dq = -dw$$
$$dS = \frac{dq}{T} = -\frac{dw}{T}$$

#### Isothermal Loading of Ideal Rubber

![](_page_40_Figure_1.jpeg)