Combining Cracks and Contact with Constitutive and Cohesive laws for Complete Calculations of Cutting

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MPM Workshop
14-15 March 2013, Salt Lake City, Utah
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and a Carnot Cycle and Caoutchouc Compression

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and a Carnot Cycle and Caoutchouc Compression
...if the Clock Consents

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Motivation

• Cutting of solid wood and wood-based composites
  • Historically done by empirical methods (Koch, 1950s)
  • Almost no analysis for wood-based composites
    • Plywood
    • Particle Board
    • Oriented Strand Board

• Wood Plastic Composites

• Should be able to do better
  • e.g., Atkins, Williams, etc.

“You can cut Trex just like regular wood.”
Experiments Too

- Build apparatus for cutting experiments
  - Based on Patel, Blackman, and Williams (from 5th ESIS TC4 meeting)
- New experiments and analysis
  - HDPE and LDPE
  - Trex (WPC)
  - Timber Tech (WPC)
  - Wood
- Theory and Numerical modeling
  - Material Point Method (MPM)
Experiments

- Four materials - HDPE, LDPE, Trex, and Timber Tech
- Rake Angles 15, 20, 22.5, 25, 30, 35, 40, 45, 50, and 55
- Depth of cut up 0.006 mm to 0.59 mm
- Semi-automatic data acquisition
- Most likely, the non-zero intercept relates to a “cutting toughness,” but how it is best determined?
Atkins Energy Analysis

Velocity = V

thickness = b

θ

T

N

F_t

F_c

α

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Atkins Energy Analysis

Griffith-Like Energy Balance (from Atkins)

\[ F_c V = \tau_y \gamma (h b V) + T \left( \frac{V \sin \phi}{\cos (\phi - \alpha)} \right) + G_c b V \]

Work = Plastic Energy + Frictional Work + Fracture Work
Griffith-Like Energy Balance (from Atkins)

\[ F_c V = \tau_y \gamma (hbV) + T \left( \frac{V \sin \phi}{\cos(\phi - \alpha)} \right) + G_c bV \]

Work = Plastic Energy + Frictional Work + Fracture Work
Williams/Atkins Analysis — Chip Force Method

- Energy balance to forces, energy release rate, yield stress, and shear angle

\[
\frac{\sigma_y}{2} \frac{h}{\sin \phi} = \left( \frac{F_c}{b} - G_c \right) \cos \phi - \left( \frac{F_t}{b} + G_c \tan \alpha \right) \sin \phi
\]

- Observations show that \( T \neq \mu N \), Williams tried:

\[
T = G_a + \mu N \quad \Rightarrow \quad \frac{F_t}{b} = \frac{Z F_c}{b} + \frac{G_a}{\cos \alpha + \mu \sin \alpha}
\]

- Minimize work (to eliminate \( \phi \))

\[
\frac{F_c}{b} = G_c + \sigma_y h \left( Z + \sqrt{1 + Z^2} + H \right)
\]

\[
H = \frac{2}{\sigma_y h} \left( \frac{G_a}{\cos \alpha + \mu \sin \alpha} + G_c (Z + \tan \alpha) \right)
\]
Sample Extrapolation

HDPE, rake angle = 55°

$G_c = 0.88 \text{ kJ/m}^2$
"Multiphysics" Numerical Modeling

Material Point Method (MPM) Modeling
1. For crack propagation
2. Cracks with cohesive zones
3. Contact capabilities
**Material Model**
- Any option implemented in code
- Used elastic/plastic with work hardening
- Have used anisotropic wood properties for trial log peeling simulations

\[ Velocity = V \]
Fracture Mechanics

- Cohesive law (cubic)

\[ \frac{\Delta \sigma}{\sigma_c} = \frac{\delta}{\delta_c} \]

- Mixed mode fracture (found 90% opening mode)

\[ \left( \frac{G_I}{G_{Ic}} \right)^n + \left( \frac{G_{II}}{G_{IIc}} \right)^n = 1 \]

\[ n, \ G_{Ic}, \ G_{IIc} \]
Contact Physics

- Tool = Rigid Material
- Coulomb Friction on chip and on bottom
- MPM Contact
  a. Contact available for “free”
  b. But needs revision to work
  c. Key is contact normals (fixed in this problem)
- Simulation output - total force on tool (same as $F_c$ and $F_t$ and new code option)
“Multiphysics” Numerical Modeling

Material Point Method (MPM) Modeling
1. For crack propagation
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Inertial Effects
- Explicit code
- Solution
  a. Ramp up tool velocity
  b. Kinetic energy “thermostat”
  c. If damping controlled, good results, otherwise failed simulation
  d. Good results gave steady state forces

Velocity = V

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MPM Contact

- Contact Detection
  - Volume Screening: \( V_{total} > V_c \)
  - Approaching: \( \Delta p_{i,a} \cdot \hat{n} < 0 \)
  - Overlap: \( \delta_i \cdot n - \delta_{contact} < 0 \)
- Normal Vector Calculation Options
  - MVG: maximum volume gradient
  - AVG: average volume gradient
  - SN: Specify the normal
- Extension of Contact to Model Interfaces

\[
T_n = D_n[u_n] \quad \text{and} \quad T_t = D_t[u_t]
\]
One Imperfect Interface Result

Challenges

1. Finding contact area for arbitrary interface orientation
2. Working with stiff interfaces $D_n, D_t \rightarrow \infty$

Model Verification

- Elastic, perfectly-plastic material
  - \( E = 1000 \) MPa, \( v = 0.33 \), \( \sigma_y = 25 \) MPa
  - Plane strain analysis

- Simple fracture law
  - \( n = 1 \), \( G_{Ic} = G_{IIc} = G_c = 2000 \) J/m\(^2\) (constant \( G_c \) regardless of mode)
  - Cubic traction law, \( \sigma_c = 40 \) MPa

- Frictionless contact

- Compare to analytical model
Simulations Problems

- Numerical difficulty resolving contact at the tool tip

Add Linear Hardening

“Elastic-plastic bending”

- Simulation Forces

Chips
Simulations vs. Plastic Bending Analysis

\[ \frac{F_c}{b \cdot G_c} \text{ vs. } \frac{\sigma_y^2 h}{2E G_c} \]

- **MPM Model**
- **Williams**
  - \( \alpha = 15^\circ, E_p = 0, E = 1000 \)

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Simulations vs. Plastic Bending Analysis

\[ \frac{F_c}{b G_c} \]

\[ \frac{\sigma_y^2 h}{2 E G_c} \]

\( \alpha = 15^\circ, E_p = 100, E' = 1000 \)

\( \alpha = 15^\circ, E_p = 0, E = 1000 \)

\( \alpha = 15^\circ, E_p = 1000, E' = 1000 \)

MPM Model

Nairn Extension
Simulations Reveal Non-Negligible Bottom Force

\[ F_t = F_0 + Z F_c \]

Semi-Analytical Model
- Revise elastic-plastic bending for \( F_b^n \)
- Insert \( F_b^n \) from simulation results

\[ T \neq G_a + \mu N \quad \text{but instead:} \quad T = \mu N \]
\[ F_b^t = \mu F_b^n \]
\[ \frac{F_t}{b} = Z \frac{F_c}{b} + (1 - \mu Z) \frac{F_b^n}{b} \]
Semi-Analytical Model

Rake = 15°, μ = 0

$\frac{F_c}{(b G_c)}$ vs. $\sigma_y^2 h/(2 E G_c)$

$E_p = 0$

$E_p = 100$
Simulations/Modeling with Friction

![Graph showing the relationship between $F_c/(bG_c)$ and $\sigma_y^2 h/(2E G_c)$ for different values of $\mu$. The graph includes data points for $\mu = 0.2$ and $\mu = 0.0$, with $E_p = 100$. The graph is labeled "Virtual Experiments."
Effect of Cohesive Stress

\[
\frac{F_c}{bG_c} = \sigma_y^2 \frac{h}{2E G_c}
\]

\[
Rake = 30^\circ, \mu = 0
\]

\[
E_p = 100
\]
Effect of Toughness

$$\sigma_{y}^{2} \frac{h}{2E G_{c}}$$

$F_{c}/(bG_{c})$

$E_{p} = 100$

$4000$

$2000$

$1000$
Effect of Rake Angle

\[ \sigma_y^2 \frac{h}{2E G_c} \]

\[ \frac{F_c}{bG_c} \]

\( \mu = 0 \)

\( \theta = 15\degree, 30\degree, 45\degree \)

\( E_p = 100 \)

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Hyperelastic-Plastic, Large Strain Material

Hyperelastic

Cumulative Plastic Strain

Hypoelastic
Hyperelastic-Plastic, Large Strain Material

Hyperelastic

Hypoelastic

Equivalent Stress
Cutting Forces

\[ \frac{F_c}{b G_c} \] vs. \[ \frac{\sigma_y^2 h}{2 E G_c} \]

- Rake 15°
- Hyperelastic
- Low Strain Isotropic
  - \( E_T = 100 \)

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In-conclusions

• HDPE, LDPE, Trex, and Timber Tech Experiments
  • Works reasonable well, but answer depends on interpretation of the $F_t$ vs. $F_c$ intercept.
  • New results for Trex and Timber Tech Wood Plastic Composites

• Numerical simulations (by MPM) are working
  • Uncertain validation
  • Potential simulations (e.g., veneer peeling) may be useful

• Forces on bottom of tool
  • How to handle it?
  • Related to sharpness
  • Essential to theory and to modeling
In-conclusions

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Carnot Cycle or an MPMgine™

- Ideal Gas as Hyperelastic Material
- Custom boundary conditions
- Trick is when to switch on returning
- Why not other materials
  - Carnot claimed general result

\[
\begin{array}{c|c}
\text{Volume (cm}^3\text{)} & \text{Pressure (MPa)} \\
\hline
150 & 0.07 \\
160 & 0.08 \\
170 & 0.09 \\
180 & 0.10 \\
190 & 0.11 \\
200 & 0.12 \\
210 & 0.13 \\
220 & 0.14 \\
230 & 0.15 \\
240 & 0.16 \\
250 & 0.17 \\
\end{array}
\]

\[
T_1 = 400K, \quad \eta = 0.122 \\
T_2 = 350K, \quad \max \eta = 0.125 = 1 - T_1/T_2
\]
Carnot Cycle on Other Materials?

- First step is cooling on isothermal expansion
  - True in coupled conduction-elasticity
  - Small effect, usually neglected
- Example
  - Tungsten with MG EOS
  - But plasticity always heats?
  - Eliminate yielding

![Tungsten Carnot Cycle Graph](chart.png)

\[
\eta = 0.0035?
\]

\[
\max \eta = 0.005 = \frac{1-T_1}{T_2}
\]

\[
T_1 = 400K
\]

\[
T_2 = 398K
\]
Ideal Rubber Elastic Material

• In 1805, John Gough described a series of experiments on caoutchouc or Indian rubber:

“For the resin evidently grows warmer the further it is extended; and the edges of the lips possess a high degree of sensibility, which enables them to discover these changes with greater facility than other parts of the body.”

• Mooney-Rivlin Hyperelastic Material

• “Ideal Rubber” from Flory

\[
\left( \frac{\partial U}{\partial L} \right)_T = 0 \quad \text{therefore} \quad dq = -dw
\]

\[
dS = \frac{dq}{T} = -\frac{dw}{T}
\]
Isothermal Loading of Ideal Rubber

\[ -T \left( \frac{\partial S}{\partial T} \right)_{T,V} \]

Elongation (mm) vs. Force (N)

Random Coil

Elongated Coil